Math 447 Homework #9 Answers

19.7 (a) Let $A = [k, +\infty)$. Let $B = [0, k+1]$. It is given that f is uniformly continuous on A, we know by Theorem 19.2 that f is uniformly continuous on B.

Suppose $\varepsilon > 0$. $\exists\, \delta_1 > 0 \ni$ if $x \in A$, $y \in A$, and $|x-y| < \delta_1$, then $|f(x)-f(y)| < \varepsilon$. $\exists\, \delta_2 > 0 \ni$ if $x \in B$, $y \in B$, and $|x-y| < \delta_2$, then $|f(x)-f(y)| < \varepsilon$. Let $\delta = \min(1, \delta_1, \delta_2)$. Suppose $x \in [0, +\infty)$, $y \in [0, +\infty)$, and $|x-y| < \delta$. Since $|x-y| < 1$, we know that either (i) x and y are both in A or (ii) x and y are both in B. In either case we may conclude that $|f(x)-f(y)| < \varepsilon$.

(b) This is immediate from (19.6) with the choice $k = 1$.

(20.17) Suppose $s_n \in (a, b)$ and $s_n \to a$. We have $f_1(s_n) \le f_2(s_n) \le f_3(s_n)$. We also have $f_1(s_n) \to L$ and $f_3(s_n) \to L$. By Problem (8.5a), we have $f_2(s_n) \to L$. Thus $\lim_{x \to a^+} f_2(x) = L$.

(19.3) Note that for $x \geq 1$ and $y \geq 1$ we have

$$\left| \frac{5x}{2x-1} - \frac{5y}{2y-1} \right| = \left| \frac{5x(2y-1) - 5y(2x-1)}{(2x-1)(2y-1)} \right|$$

$$= 5 \frac{|y-x|}{(2x-1)(2y-1)} \leq 5|y-x|, \quad \text{since } \begin{array}{l} 2x-1 \geq 1 \\ 2y-1 \geq 1 \end{array}$$

Given ε, set $\delta = \varepsilon/5$. Then if $x \geq 1$, $y \geq 1$, and $|x-y| < \delta$, the calculation above shows that

$$\left| \frac{5x}{2x-1} - \frac{5y}{2y-1} \right| < 5\delta = \varepsilon.$$

Thus $f(x)$ is uniformly continuous on $[1, +\infty)$.