MATH 312, FALL 2002 - PROBLEM SET 1

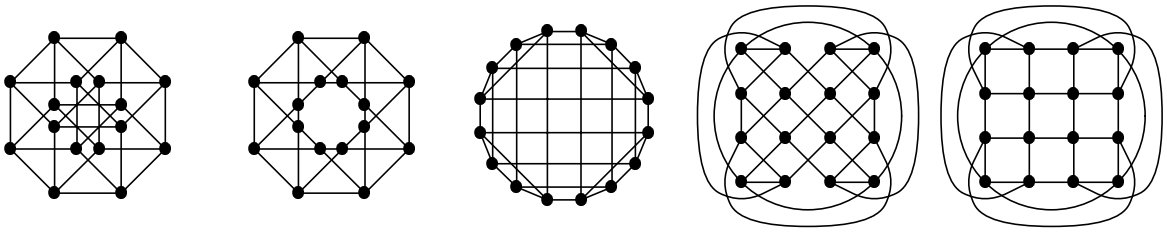
WARMUP PROBLEMS: Section 1.1: #2, 4, 5, 8, 9, 10, 33. Do not write these up! Just think about how to solve them to make sure you understand the material before working on the written homework.

OTHER INTERESTING PROBLEMS: Section 1.1: #25, 34, 35, 38, 40, 45. Do not write these up! If you have time after doing the homework, you might think about these for extra practice.

WRITTEN PROBLEMS: Solve five of the following six problems (graduate students registered for one unit credit must do all six problems). Due Friday, September 6 (problem sets will generally be due on Wednesdays, with solution sets distributed on Fridays).

Words like "construct", "show", "obtain", "determine", etc., explicitly state that proof is required. Full credit for solutions to most problems requires proof of the statements made. Use *sentences*; you cannot give a proof without words. Results covered in class can be used without proof (just state them correctly).

1. For each graph below, determine whether it is bipartite. Determine also which pairs of these graphs are isomorphic. (Partial credit is available for correct parts of answers.)



2. Determine whether the Petersen graph is bipartite, and find the size of its largest independent set.

3. Let G be a graph with girth 4 in which every vertex has degree k. Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.

4. Prove that a self-complementary graph with n vertices exists if and only if n or $n-1$ is divisible by 4. (Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For $n \equiv 1 \bmod 4$, add one vertex to the graph constructed for $n-1$.)

5. Prove that a graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.

6. Prove that every simple graph with at least two vertices has two vertices of equal degree. Is the conclusion true for graphs in general? What about loopless graphs?