

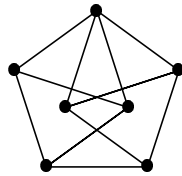
MATH 312, FALL 2002 - PROBLEM SET 10

WARMUP PROBLEMS: Section 5.1: #3, 4, 8, 9, 11, 15. Do not write these up! Think about these to make sure you understand the material.

OTHER INTERESTING PROBLEMS: Section 5.1: #22, 23, 29, 32, 35. Do not write up! Think about these if you have time.

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Nov. 13.

1. Prove that if G is bipartite and $\delta(G) \geq 1 + \frac{1}{4}n(G)$, then $\kappa'(G) = \delta(G)$.
2. Compute the clique number, the independence number, and the chromatic number of the graph below. Does either bound in Proposition 5.1.7 prove optimality for some proper coloring? Is the graph color-critical?



3. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.
4. Prove that a graph G is m -colorable if and only if $\alpha(G \square K_m) \geq n(G)$.
5. Prove that every graph G has a vertex ordering relative to which greedy coloring uses $\chi(G)$ colors.
6. Prove that $\chi(G) = \omega(G)$ when \overline{G} is bipartite. (Hint: Phrase the claim in terms of \overline{G} and apply results on bipartite graphs.)