MATH 312, FALL 2002 - PROBLEM SET 12

WARMUP PROBLEMS: Section 6.1: #2, 3, 4, 5, 7. Section 6.2: 1, 4. Do not write these up! Think about these to make sure you understand the material.

OTHER INTERESTING PROBLEMS: Section 6.1: #10, 12, 13, 17, 20, 27, 30, 31. Section 6.2: #7, 8, 12. Do not write up! Think about these if you have time.

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Dec. 4.

1. Prove that a set of edges in a connected plane graph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*.

2. Prove that every n-vertex plane graph isomorphic to its dual has $2n-2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

3. For $n \geq 2$, determine the maximum number of edges in a simple outerplane graph with n vertices, giving three proofs.

a) By induction on n.

b) By using Euler's Formula.

c) By adding a vertex in the unbounded face and using Theorem 6.1.23.

4. The *rhombicosadodecahedron* is a polyhedron in which every vertex is incident to one triangular face, one pentagonal face, and two (opposite) quadrilateral faces. Determine the number of faces in the rhombicosadodecahedron. (Comment: The toy construction system "Zometool" is based on this polyhedron.)

5. Give three proofs that the Petersen graph is nonplanar.

a) Using Kuratowski's Theorem.

b) Using Euler's Formula and the fact that the Petersen graph has girth 5.

c) By proving that the Petersen graph is contractible to K_5.

6. Let H be a graph with maximum degree at most 3. Prove that a graph G contains a subdivision of H if and only if G contains a subgraph contractible to H.