MATH 312, FALL 2002 - PROBLEM SET 2

WARMUP PROBLEMS: Section 1.2: #1, 2, 8, 26, 33. Section 1.3: #1, 2, 4. Do not write these up! Think about how to solve them to make sure you understand the material before doing the homework.

OTHER INTERESTING PROBLEMS: Section 1.2: #20, 26, 27, 37. Section 1.3: #9, 12, 17, 24, 25, 30. Do not write these up! These are interesting problems (related to what we have discussed) to provide extra practice.

WRITTEN PROBLEMS: Do five of the six problems below (graduate students registered for one unit credit must do all six problems). Due Wednesday, September 11.

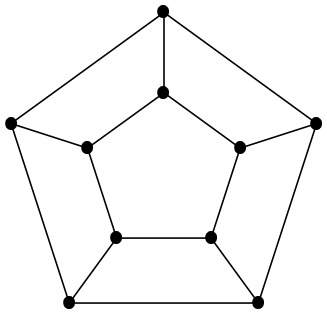
1. Prove or disprove each of the following statements:

a) Every Eulerian bipartite graph has an even number of edges.

b) Every Eulerian simple graph with an even number of vertices has an even number of edges.

c) If G is an Eulerian graph with edges e and f that share a vertex, then G has an Eulerian circuit in which e and f appear consecutively.

2. In the graph below, find a bipartite subgraph with the maximum number of edges. Prove that this is the maximum, and determine whether this is the only bipartite subgraph with this many edges.



3. Prove that every n-vertex graph with at least n edges contains a cycle.

4. Let P and Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex. Is it true that P and Q must have a common edge (give a proof or a counterexample)?

5. For $k \geq 2$, prove that a k-regular bipartite graph has no cut-edge.

6. Use counting arguments with graphs for both problems below.

a) Count the 6-cycles in $K_{m,n}$.

b) Prove that $\binom{n}{2} = \binom{k}{2} + k(n-k) + \binom{n-k}{2}$ for $0 \leq k \leq n$.