MATH 312, FALL 2002 - PROBLEM SET 5

WARMUP PROBLEMS: Section 2.2: #1, 3, 4. Section 2.3: #2, 3, 5. Section 3.1 #2,3. Do not write these up! Think about how to clarify your understanding.

OTHER INTERESTING PROBLEMS: Section 2.2: #5, 7, 8, 15, 33. Section 2.3: #6, 7, 10, 14. Section 3.1 #9, 10. Do not write these up! Think about some of these if you have time.

1. Let G be the 3-regular graph with $4m$ vertices formed from m pairwise disjoint kites by adding m edges to link them in a ring, as shown on the right above for $m = 6$. Prove that $\tau(G) = 2m8^m$.

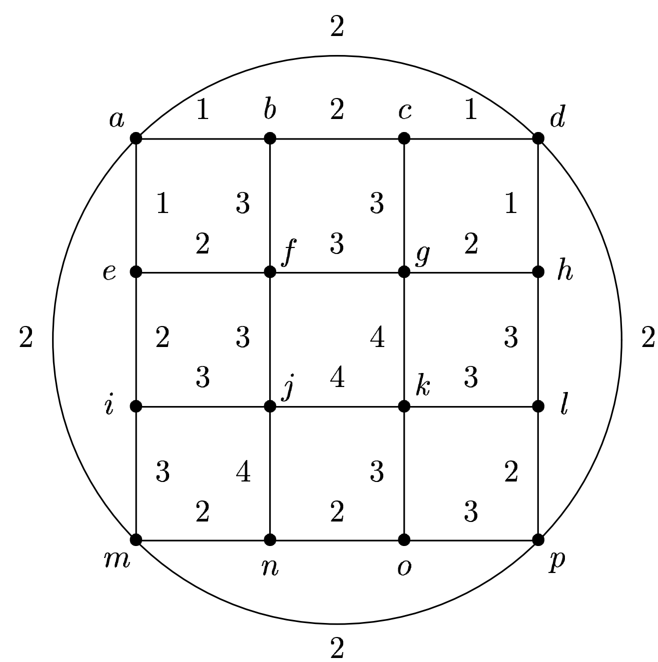
2. Count the following sets of trees with vertex set $[n]$, giving two proofs for each: one using the Prüfer correspondence and one by direct counting arguments.

a) trees that have 2 leaves.

b) trees that have $n - 2$ leaves.

3. Compute $\tau(K_{2,m})$. Also compute the number of isomorphism classes of spanning trees of $K_{2,m}$.

4. Use Prim's Algorithm and Kruskal's Algorithm to find minimum spanning trees in the weighted graph below (show the sequence in which edges are added).



5. Interpreting edge weights as lengths in the graph above, find a shortest path from b to o using Dijkstra's Algorithm. Show in order the sequence of vertices x for which you find the distance from b to x.

6. Prove or disprove: Every tree has at most one perfect matching.