MATH 312, FALL 2002 - PROBLEM SET 6

WARMUP PROBLEMS: Section 3.1: #1, 3, 4, 5, 6. Section 3.2: #3. Do not write these up! Think about them to clarify your understanding.

OTHER INTERESTING PROBLEMS: Section 3.1: #10, 18, 19, 25, 30, 38, 44. Do not write these up!

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Oct. 16.

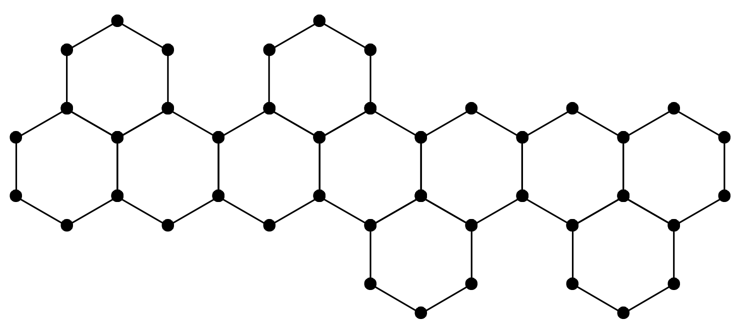
1. Prove that every maximal matching in a graph G has at least $\alpha'(G)/2$ edges.

2. Let G be an X, Y-bigraph such that $|N(S)| > |S|$ whenever $\varnothing \neq S \subset X$. Prove that every edge of G belongs to some matching that saturates X.

3. A *permutation matrix* P is a 0,1-matrix having exactly one 1 in each row and column. Prove that a square matrix of nonnegative integers can be expressed as the sum of k permutation matrices if and only if all row sums and column sums equal k.

4. A deck of mn cards with m values and n suits consists of one card of each value in each suit. The cards are dealt into an n-by-m array. Prove that there is a set of m cards, one in each column, having distinct values.

5. Exhibit a perfect matching in the graph below or give a short proof that it has none.



6. Use the König–Egerváry Theorem to prove that every bipartite graph G has a matching of size at least $e(G)/\Delta(G)$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.