

## MATH 312, FALL 2002 - PROBLEM SET 7

TEST #2: The second test will be Thursday October 24 from 6 to 7:30 in 341 Altgeld Hall or from 7 to 8:30 in 343 Altgeld Hall.

WARMUP PROBLEMS: Section 3.2: #3. Section 3.3 #1, 3. Section 4.1 #1, 2, 3, 4. Do not write these up! Think about them to make sure you understand the material.

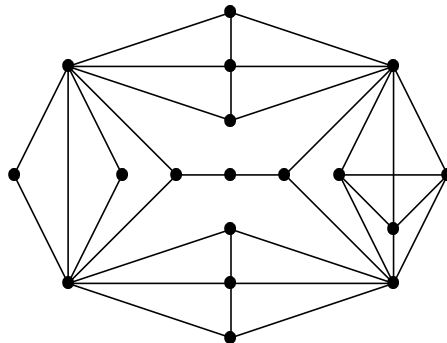
OTHER INTERESTING PROBLEMS: Section 3.2: #9, 12. Section 3.3: #6, 8, 14, 15, 16, 24, 26. Section 4.1 #8, 9, 10. Do not write these up!

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Oct. 23.

1. Determine the stable matchings resulting from the Proposal Algorithm run with men proposing and with women proposing, given the preference lists below.

Men $\{u, v, w, x, y, z\}$	Women $\{a, b, c, d, e, f\}$
$u : a > b > d > c > f > e$	$a : z > x > y > u > v > w$
$v : a > b > c > f > e > d$	$b : y > z > w > x > v > u$
$w : c > b > d > a > f > e$	$c : v > x > w > y > u > z$
$x : c > a > d > b > e > f$	$d : w > y > u > x > z > v$
$y : c > d > a > b > f > e$	$e : u > v > x > w > y > z$
$z : d > e > f > c > b > a$	$f : u > w > x > v > z > y$

2. Exhibit a maximum matching in the graph below, and use a result in this section to give a short proof that it has no larger matching.



3. For each  $k > 1$ , construct a  $k$ -regular simple graph having no 1-factor.
4. For every graph  $G$ , prove that  $\beta(G) \leq 2\alpha'(G)$ . For each  $k \in \mathbb{N}$ , construct a simple graph  $G$  with  $\alpha'(G) = k$  and  $\beta(G) = 2k$ .
5. Let  $G$  be an  $X, Y$ -bigraph. Let  $H$  be the graph obtained from  $G$  by adding one vertex to  $Y$  if  $n(G)$  is odd and then adding edges to make  $Y$  a clique.
  - a) Prove that  $G$  has a matching of size  $|X|$  if and only if  $H$  has a 1-factor.
  - b) Prove that if  $G$  satisfies Hall's Condition ( $|N(S)| \geq |S|$  for all  $S \subseteq X$ ), then  $H$  satisfies Tutte's Condition ( $o(H - T) \leq |T|$  for all  $T \subseteq V(H)$ ).
  - c) Use parts (a) and (b) to obtain Hall's Theorem from Tutte's Theorem.
6. A graph  $G$  is *factor-critical* if each subgraph  $G - v$  obtained by deleting one vertex has a 1-factor. Prove that  $G$  is factor-critical if and only if  $n(G)$  is odd and  $o(G - S) \leq |S|$  for all nonempty  $S \subseteq V(G)$ .