MATH 312, FALL 2002 - PROBLEM SET 8

WARMUP PROBLEMS: Section 4.1: #5, 6, 7. Section 4.2: #2, 4, 5. Do not write these up! Think about how to solve them to make sure you understand the material.

OTHER INTERESTING PROBLEMS: Section 4.1: #11, 19, 20, 24, 25, 34. Section 4.2: #13, 14, 19. Do not write these up! Think about some of these if you have time.

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Oct. 30.

1. Let n, k be positive integers with n even, k odd, and $n > k > 1$. Let G be the k-regular simple graph formed by placing n vertices on a circle and making each vertex adjacent to the opposite vertex and to the $(k-1)/2$ nearest vertices in each direction. Prove that $\kappa(G) = k$.

2. Let G be a connected graph in which for every edge e, there are cycles C_1 and C_2 containing e whose only common edge is e. Prove that G is 3-edge-connected. Use this to show that the Petersen graph is 3-edge-connected.

3. Let G be an r-connected graph of even order having no $K_{1,r+1}$ as an induced subgraph. Prove that G has a 1-factor.

4. Prove that the symmetric difference of two different edge cuts is an edge cut. (Hint: Draw a picture illustrating the two edge cuts and use it to guide the proof.)

5. Prove that a connected graph is k-edge-connected if and only if each of its blocks is k-edge-connected.

6. For a connected graph G with at least three vertices, prove that the following statements are equivalent.

A) G is 2-edge-connected.
B) Every edge of G appears in a cycle.
C) G has a closed trail containing any specified pair of edges.
D) G has a closed trail containing any specified pair of vertices.