

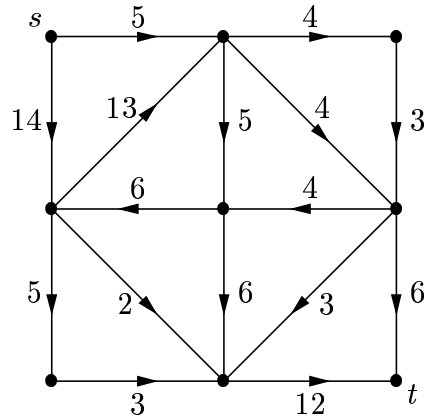
MATH 312, FALL 2002 - PROBLEM SET 9

WARMUP PROBLEMS: Section 4.2: #2, 3, 4, 5. Section 4.3 #1. Do not write these up! Think about how to solve them to make sure you understand the material.

OTHER INTERESTING PROBLEMS: Section 4.2: #6, 7, 8, 23, 26. Section 4.3: #2, 12, 14. Do not write these up! Think about these if you have time.

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, Oct. 24.

1. In the network below, find a maximum flow from s to t . Prove that your answer is optimal by using the dual problem, and explain why this proves optimality.



2. Use network flows to prove Menger's Theorem for edge-disjoint paths in graphs: $\kappa'(x, y) = \lambda'(x, y)$. (Hint: Use the second transformation suggested in Remark 4.3.15.)

3. Use network flows to prove the König–Egerváry Theorem ($\alpha'(G) = \beta(G)$ if G is bipartite).

4. Several companies send representatives to a conference; the i th company sends m_i representatives. The organizers of the conference conduct simultaneous networking groups; the j th group can accommodate up to n_j participants. The organizers want to schedule all the participants into groups, but the participants from the same company must be in different groups. The groups need not all be filled. Show how to use network flows to test whether the constraints can be satisfied.

5. Let G be a k -connected graph, and let S, T be disjoint subsets of $V(G)$ with size at least k . Prove that G has k pairwise disjoint S, T -paths.

6. Determine $\kappa(u, v)$ and $\kappa'(u, v)$ in the graph drawn below.

