Course Description - FALL 2002

MATH 475

The Probabilistic Method

Class hours: 3pm-3:50pm MWF
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Overview:

The probabilistic method proves the existence of an object with a given property by showing that the property occurs with non-zero probability when we select a random object from an appropriate probability space. Using the probabilistic method we can prove theorems that themselves involve no probability. This way we can often avoid complicated constructions. Further study of the probabilistic method leads naturally to the study of random structures, e.g. the random graph.

The topics being covered in the class will concentrate on basic methods as well as illustration of important applications in combinatorics as well as other areas of mathematics.

The proposed topics to be covered include (but are not limited to):

1 The basic method, Ramsey numbers, sum-free subsets, hypergraph 2-coloring, dominating sets in graphs and edge connectivity.

2 Linearity of expectation, Hamiltonian paths in tournaments and the conjecture of Szele, Minc Conjecture.

3 The second moment method, Turán's theorem on the number of prime factors of integers, random graphs and threshold functions, cliques in random graphs.

4 Alterations: Graphs with high girth and high chromatic number, improved hypergraph 2-coloring, bounding of large deviations and consistent arcs in tournaments.

5 The local lemma and its symmetric version, directed cycles, linear arboricity of graphs.

6 Correlation Inequalities: the four function theorem, the FKG inequality, Kleitman's Theorem.

7 Martingales, concentration inequalities, the chromatic number of random graphs.

8 Poisson approximation, the Janson inequalities, counting subgraphs in random graphs.

9 Thresholds, branching processes, 0-1 laws.

10 Applications to computer science: circuit complexity, codes.

11 Geometric applications: the VC dimension of a range space and its applications.

12 Pseudo-Randomness: expanders, quasi-random graphs, regularity lemma, The Rödl Nibble.

Prerequisites: Math 470 or Math 312. Previous exposure to probability and/or analysis will be helpful but not required.

Text: Our main source will be *"The Probabilistic Method"* (2nd edition) by Noga Alon and Joel Spencer. We will also use other books and papers.