Math 225, Fall 2011
Coordinate System Worksheet
October 27, 2011

We represent vectors in $\mathbb{R}^n$ in a canonical way as n-tuples of real numbers. This is actually choosing a distinguished basis $\mathbf{e}_1, \ldots, \mathbf{e}_n$ and then writing a vector $\mathbf{v} = \sum_{k=1}^{n} \alpha_k \mathbf{e}_k$ for some uniquely determined scalars $\alpha_1, \ldots, \alpha_n$. The scalars α_i are the *coordinates* of $\mathbf{v}$ in terms of the basis $E = \{\mathbf{e}_, \ldots, \mathbf{e}_n\}$. We denote this by $\mathbf{v} = \mathbf{v}^E = [\alpha_k]^E$. But we can change the basis to some other basis $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ and then the coordinates would change so that $\mathbf{v}^B = [\beta_k]^B$ i.e. $\mathbf{v} = \sum_{k=1}^{n} \beta_k \mathbf{b}_k$. When we are using the canonical basis E, we often drop the superscript E in the notation.

Question: How do we determine $[\beta_k]_B$ knowing $\mathbf{v}$ and B?

Answer: Let A be the $n \times n$ matrix $[\mathbf{b}_1 | \ldots | \mathbf{b}_n]$. Then $A\mathbf{v}^B = \mathbf{v}$.

1) Take the basis $B = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix}\}$, and the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and use linear equations to solve for β_1, β_2 and so find $\mathbf{v}^B = [\beta_k]^B$.

2) Confirm that the formula $A\mathbf{v}^B = \mathbf{v}$ works in this case.

3) Take another basis $C = \left\{ \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$. Find a 2×2 matrix D such that $D\mathbf{v}^C = \mathbf{v}^B$.

4) Confirm that your matrix D from 3) works by finding $\mathbf{v}^C$ explicitly and checking that $D\mathbf{v}^C = \mathbf{v}^B$.