1	2	3	4	5	TOT

Math 225, Spring 2010
Midterm IB
February 18, 2010

NAME KEY

A) DO NOT OPEN YOUR EXAM UNTIL YOU ARE TOLD TO DO SO!

B) Be sure to write your name on this page.

C) This is a closed book, closed notes exam. No calculators allowed.

D) No cell phones allowed. Turn them OFF before coming to the exam room. If you are seen with a cell phone in hand during the exam or if your cell phone is heard, it will be considered cheating and you will be asked to leave.

E) No other electronic devices allowed e.g. MP3 players, PDAs, etc. Same rules as cell phones.

F) If you have a question, raise your hand and a proctor will come to you. Once you stand up, you are done with the exam. You will not be permitted to leave the room and return during the exam.

G) Every exam is worth a total of 100 points. Each exam has 6 pages including the cover sheet. Check to see that you have all the pages.

H) If you finish early, please hand in your exam and leave quietly in consideration of your fellow students. You need to show a picture ID with a clear picture when you turn in your exam. When time is up, you will be instructed to put down your writing utensil and close your exam.

1) (20pts) Consider the system of linear equations

$$\frac{2}{3}x + \frac{1}{3}y = 2 \qquad I$$
$$\frac{1}{3}x + \frac{1}{2}y = 1 \qquad II$$

a) This system either has no solutions, one solution, or many solutions. Which is the case here?
b) In case there are solutions to this system, describe all of them

$II \quad \frac{2}{3}x + y = 2$

$I - II: \quad -\frac{2}{3}y = 0 . \Rightarrow y = 0, \;\&\; x = 3$

a) There is only one solution

b) It is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

2) (20pts) Let

$$A = \begin{array}{cc} \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 0 & 2 & 1 \end{bmatrix} & \begin{array}{l} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array} \end{array}$$

Showing all row operations, find the row echelon form for this matrix.

$$A \longrightarrow \begin{array}{cc} \begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & -2 & 4 & -3 \end{bmatrix} & \begin{array}{l} \mathrm{I} \\ \mathrm{II} - \mathrm{I} \\ \mathrm{III} - 2\mathrm{I} \end{array} \end{array}$$

$$\longrightarrow \begin{array}{cc} \begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 12 & 1 \end{bmatrix} & \begin{array}{l} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} + 2\mathrm{II} \end{array} \end{array}$$

row echelon form

3) (20pts) Consider the system of linear equations

$$2\,x - y + z + w = 1$$

$$x + y + 2\,z - w = -1$$

a) Using free variables, find the general solution of this system.

b) How many free variables are there in this general solution?

$\boxed{\text{two}}$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & 1 & 1 \\ 1 & 1 & 2 & -1 & -1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & -1 \\ 0 & -3 & -3 & 3 & 3 \end{array}\right] \begin{array}{l} II \\ I - 2II \end{array}$$

$x + y + 2z - w = -1$

$y + z - w = -1$

$\boxed{z, w \text{ free}}$

$y = -1 + w - z$

$x = -1 + w - 2z - y = -1 + w - 2z - (-1 + w - z)$

$= -z$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

4) (20pts) Consider the 2×2 matrices A, B, C, X. Assume that

$$2X + A = C - \ BX.$$

a) Solve for X in terms of A, B, C and their inverses. You might not need to use all of these matrices in your formula.

b) Find X if

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 3 \\ 1 & -1 \end{bmatrix}, \ C = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}.$$

a) $2IX + BX = C - A$

$(2I + B)X = C - A$

$X = (2I + B)^{-1}(C - A)$

b) $2I + B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ $\quad \det(2I + B) = -1$

$2I + B = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

$X = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} \left(\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right)$

$= \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} \left(\begin{bmatrix} -2 & 2 \\ -1 & 0 \end{bmatrix} \right)$

$= \begin{bmatrix} -1 & -2 \\ 0 & 2 \end{bmatrix}$

5) (20pts) Consider the matrix A given by

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Compute the inverse matrix A^{-1} of A showing all your steps in the calculation.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 3 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \text{II} - \text{I}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 4 & 3 & -1 & 1 & 0 \end{array}\right] \begin{array}{c} \text{III} \\ \text{II} \end{array} \to \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & -4 \end{array}\right]. \quad \text{III} - 4\text{II}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & -4 \end{array}\right] \text{I} + 2\text{II}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -6 \\ 0 & 1 & 0 & -1 & 1 & -3 \\ 0 & 0 & 1 & 1 & -1 & 4 \end{array}\right] \begin{array}{l} \text{I} + 2\,\text{III} \\ \text{II} + \text{III} \\ -\text{III} \end{array}$$

$$\underbrace{\begin{bmatrix} -1 & 2 & -6 \\ -1 & 1 & -3 \\ 1 & -1 & 4 \end{bmatrix}}_{A^{-1}}$$

Check $AA^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & -6 \\ -1 & 1 & -3 \\ 1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{-1}A = \begin{bmatrix} -1 & 2 & -6 \\ -1 & 1 & -3 \\ 1 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$