Sample Exam Problems for Math 225 Fall 2011

Sample Cover Page

A) DO NOT OPEN YOUR EXAM UNTIL YOU ARE TOLD TO DO SO!

B) Be sure to write your name on this page.

C) This is a closed book, closed notes exam. No calculators allowed.

D) No cell phones allowed. Turn them OFF before coming to the exam room. If you are seen with a cell phone in hand during the exam or if your cell phone is heard, it will be considered cheating and you will be asked to leave.

E) No other electronic devices allowed e.g. MP3 players, PDAs, etc. Same rules as cell phones.

F) If you have a question, raise your hand and a proctor will come to you. Once you stand up, you are done with the exam. You will not be permitted to leave the room and return during the exam.

G) Every exam is worth a total of 100 points. Each exam has 6 pages including the cover sheet. Check to see that you have all the pages.

H) If you finish early, please hand in your exam and leave quietly in consideration of your fellow students. You need to show a picture ID with a clear picture when you turn in your exam. When time is up, you will be instructed to put down your writing utensil and close your exam.

1) Consider the system of linear equations

$$\frac{1}{3}x + \frac{2}{3}y = 2$$
$$\frac{1}{2}x + \frac{1}{3}y = 1$$

a) This system either has no solutions, one solution, or many solutions. Which is the case here?
b) In case there are solutions to this system, describe all of them

2) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 2 & 1 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

Showing all row operations, find the row echelon form for this matrix.

3) Consider the system of linear equations

$$2\,x - y + z + w = 1$$

$$x + y - z + 2\,w = -1$$

a) Using free variables, find the general solution of this system.

b) How many free variables are there in this general solution?

4) Consider the 3×3 matrix

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 1 & 0 \\ 2 & 3 & -2 \end{bmatrix}$$

. Decide if the column vectors of this matrix are linear independent and if they are not give an explicit linear relationship between them to show that they are linear dependent.

5) Consider the 2×2 matrices A, B, C, X. Assume that

$$2X + A = B - CX.$$

a) Solve for X in terms of A, B, C and their inverses. You might not need to use all of these matrices in your formula.

b) Find X if

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 3 \\ 1 & -1 \end{bmatrix}.$$

6) Consider the matrix A given by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Compute the inverse matrix A^{-1} of A showing all your steps in the calculation.

7) Consider the 2×2 matrices

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, C = \begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix}.$$

a) Compute $D = AB^{-1}C$.

b) Show by a direct computation that $\det D = \frac{\det A \det C}{\det B}$.

8) Let a, b, c be real numbers. Consider the 3×3 matrix

$$A = \begin{bmatrix} a & 2 & 3 \\ b & 2 & 1 \\ c & 2 & 5 \end{bmatrix}.$$

a) What is the formula for the determinant of A in terms of a, b, c?
b) Give an example of a, b, c for which A is not invertible and give an example of a, b, c for which A is invertible.

9) Consider the points in the plane: $(-1,5),(3,9),(1,4),(5,8)$.
a) These points determine a quadrilateral. Why is it a parallelogram?
b) Using determinants, find the area of this parallelogram.

10) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 1 \\ -1 & 4 & 5 & 1 \end{bmatrix}.$$

a) Find a basis for the column space of A.
b) What is the dimension of the column space of A?

11) Consider the matrix

$$B = \begin{bmatrix} 3 & 2 & 3 & 4 \\ 1 & 2 & 1 & 3 \\ 2 & 0 & 2 & 1 \end{bmatrix}.$$

a) Find a basis of the null space of B.
b) What is the dimension of the null space of B?

12) Consider the 3×3 matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 2 & 1 & 0 \\ -2 & 4 & -2 \end{bmatrix}.$$

Using the explicit values of the entries of the matrix, find the characteristic equation for this matrix.

13) Consider the 2×2 matrix $\begin{bmatrix} -2 & 2 \\ 4 & 1 \end{bmatrix}$.

a) Find the eigenvalues of this matrix and find an eigenvector for each of the eigenvalues.

b) Is there a basis of eigenvectors?

14) Consider the 2×2 matrix $B = \begin{bmatrix} 7 & 5 \\ -10 & -8 \end{bmatrix}$. This matrix has distinct eigenvalues and there is a basis of $\mathbb{R}^2$ consisting of eigenvectors of B.

a) Find the eigenvalues and at least one eigenvector for each of the eigenvalues of this matrix.

b) Using the eigenvectors in a), find a 2×2 matrix P such that $P^{-1}\,B\,P$ is a diagonal matrix.

15) Consider the matrix $B = \begin{bmatrix} 24 & -50 \\ 10 & -21 \end{bmatrix}$. This matrix has distinct eigenvalues and there is a basis of $\mathbb{R}^2$ consisting of eigenvectors of B.

a) Find the two eigenspaces of B.

b) Find the cosine of the angle between the eigenspaces of B.

16) Consider the vectors $\mathbf{v} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Find a scalar c such that $\mathbf{v}$ and $\mathbf{w} - c\mathbf{v}$ are perpendicular to one another.