

# Math 231 CD3 Worksheet 9

April 21, 2010

## Parametric Equations

1. Eliminate  $\theta$  to find a Cartesian equation of the curve given by  $x = 4 \cos \theta$ ,  $y = 5 \sin \theta$ ,  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ . Then sketch a curve and indicate with an arrow the direction in which the curve is traced as the parameter increases
2. Compare the curves represented by the parametric equations. How do they differ?  
(a)  $x = t^3$ ,  $y = t^2$  (b)  $x = e^{-3t}$ ,  $y = e^{-2t}$  (c)  $x = t^6$ ,  $y = t^4$ .
3. If  $x = t^3 - 12t$ , and  $y = t^2 - 1$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . For which values of  $t$  is the curve concave up?.
4. If  $x = \sin 2t \cos t$ ,  $y = \sin 2t \sin t$  then how many different tangents are there at  $(0, 0)$ ?
5. Set up an integral that represents the length of the curve  $x = t - t^2$ ,  $y = \frac{4}{3}t^{3/2}$ ,  $1 \leq t \leq 2$ .
6. Find the length of  $x = e^t \cos t$ ,  $y = e^t \sin t$ ,  $0 \leq t \leq \pi$ .
7. Find the distance travelled by the particle given by parametric equations  $x = \cos^2 t$ ,  $y = \cos t$ , as  $t$  ranges from 0 to  $4\pi$ . Compare with the length of the curve.
8. If  $x = 3t - t^3$  and  $y = 3t^2$  for  $0 \leq t \leq 1$ , find the exact area of the surface obtained by rotating the given curve about the  $x$ -axis.
9. If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$  for  $0 \leq \theta \leq \frac{\pi}{2}$ , find the exact area of the surface obtained by rotating the given curve about the  $x$ -axis.