SOLUTIONS TO SELECTED PROBLEMS IN SECTION 5.2

4. $\begin{vmatrix} 5-\lambda & -3 \\ -4 & 3-\lambda \end{vmatrix} = (5-\lambda)(3-\lambda) - 12 = \lambda^2 - 8\lambda + 3$. Use the quadratic formula to get eigenvalues $4 \pm \sqrt{13}$.

13. Need to find $\begin{vmatrix} 6-\lambda & -2 & 0 \\ -2 & 9-\lambda & 0 \\ 5 & 8 & 3-\lambda \end{vmatrix}$. Making use of the third column, we get

$$\begin{vmatrix} 6-\lambda & -2 & 0 \\ -2 & 9-\lambda & 0 \\ 5 & 8 & 3-\lambda \end{vmatrix} = (3-\lambda)\{(6-\lambda)(9-\lambda) - 4\} = (3-\lambda)(5-\lambda)(10-\lambda)$$

from which we get eigenvalues $\{3, 5, 10\}$.

16. Use the fact that it is lower-triangular: eigenvalues are $\{5, -4, 1\}$, where 1 is a double root.

19. The given formula is true for any λ. Plug in $\lambda = 0$. Note that we have to count multiplicities. For example, if a 4×4 matrix has $(\lambda-2)(\lambda-2)(\lambda+3)(\lambda-4)$ as its characteristic polynomial, then the determinant is **NOT** $2 \cdot (-3) \cdot 4 = -24$, **BUT** $2 \cdot 2 \cdot (-3) \cdot 4 = -48$.