SOLUTIONS TO SELECTED PROBLEMS IN SECTION 2.3

1. Invertible, since $\det A = -9 \neq 0$.

2. Not invertible, since $\det A = 0$. Another argument: columns are multiples of each other, so they are linearly dependent.

6. Not invertible. $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ and this shows that the matrix has only two pivot positions.

11. Note that in these statements, we are assuming that A is an $n \times n$ matrix, as mentioned in the beginning of exercises.
(a) True. See the equivalence between (b) and (d) in p.129.
(b) True. See the equivalence between (h) and (e) in p.129.
(c) False. Consider the zero matrix in place of A and any nonzero vector $\vec{b}$. Remark: here A is not assumed to be invertible, so (g) in p.129 is not applicable.
(d) True. If A had n pivot positions, then it would mean that the system $A\vec{x} = \vec{0}$ has only the trivial solution by the equivalence between (c) and (d).
(e) True. I told you that if A is invertible then A^T is invertible (and vice versa) and $(A^T)^{-1} = (A^{-1})^T$.

15. No, because columns of A are linearly dependent.

17. If A is invertible, then so is A^{-1} with $(A^{-1})^{-1} = A$. Use the equivalence of (a) and (e) in p.129 applied to A^{-1}.

19. The system has the unique solution $\vec{x} = D^{-1}\vec{b}$ for each 7-vector $\vec{b}$.

27. You can show that A is right-invertible, and hence invertible.

28. You can show that B is left-invertible, and hence invertible.