

SOLUTIONS TO SELECTED PROBLEMS IN SECTION 4.2

2. Just compute: $A\vec{w} = \begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This calculation shows that \vec{w} is in Nul A .

3. $A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix} \xrightarrow{R_1 \mapsto R_1 - 3R_2} \begin{bmatrix} 1 & 0 & -7 & 6 \\ 0 & 1 & 4 & -2 \end{bmatrix}$, giving

$$\begin{cases} x_1 = 7x_3 - 6x_4 \\ x_2 = -4x_3 + 2x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

as the solution set of the homogeneous system associated with A . The solution set can be

expressed as the set of all 4-vectors of the form $\begin{bmatrix} 7x_3 - 6x_4 \\ -4x_3 + 2x_4 \\ x_3 \\ x_4 \end{bmatrix}$, or, equivalently, the solu-

tion set is $\text{Span} \left\{ \begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

9. W is a vector space because $W = \text{Nul } A$, where $A = \begin{bmatrix} 1 & -2 & -4 & 0 \\ 2 & 0 & -1 & -3 \end{bmatrix}$. In another way, you may check the conditions in p.220, but that would be harder and longer.

16. Given set can be written as $\left\{ b \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -1 \\ 1 \\ 5 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix} : b, c, d \text{ real} \right\}$ and this is

another expression for $\text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix} \right\}$. So one can take, for example,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$

22. Referring to problem 3 above, we see that $\begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix}$ can be an example of nonzero vectors in $\text{Nul}A$. Any column, say $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, of A sits in $\text{Col}A$.

24. The augmented matrix $\begin{bmatrix} -8 & -2 & -9 & \vdots & 2 \\ 6 & 4 & 8 & \vdots & 1 \\ 4 & 0 & 4 & \vdots & -2 \end{bmatrix}$ reduces to $\begin{bmatrix} 1 & 0 & 1 & \vdots & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$. It is very easy to make computational mistakes when you perform row operations on a big matrix like this. If you are not sure whether your calculation is correct or not, using *Mathematica* or other similar software could be a nice solution and that is what I do when I check my calculation. In this example, the *Mathematica* command I used was

```
m = {{-8, -2, -9, 2}, {6, 4, 8, 1}, {4, 0, 4, -2}}
RowReduce[m]
```

and the software gave

```
{{1, 0, 1, -1/2}, {0, 1, 1/2, 1}, {0, 0, 0, 0}}
```

in return. Going back to the problem, the rightmost column of the augmented matrix is not pivotal, which means the vector \vec{w} is in the column space of A . To determine if w is in $\text{Nul}A$, simply compute the product $A\vec{w}$ which turns out to be $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This shows that \vec{w} is in $\text{Nul}A$.

25. a. True.

b. False. If A were the $m \times n$ zero matrix, then true.

d. False. If the equation is consistent, what we can say is that \vec{b} is in $\text{Col}A$. If we can assume that the equation is consistent for all \vec{b} in \mathbb{R}^n , then the answer would be True.

f. True. If $A = \begin{bmatrix} | & | & \cdots & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ | & | & \cdots & | \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, then $A\vec{x} = x_1\vec{v}_1 + x_2\vec{v}_2 + \cdots + x_n\vec{v}_n$.

Therefore, $\text{Col}A = \text{Span}\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\} = \{A\vec{x} : \vec{x} \in \mathbb{R}^n\}$.

27. The system can be written as $A\vec{x} = \vec{0}$, where $A = \begin{bmatrix} 1 & -3 & -3 \\ -2 & 4 & 2 \\ -1 & 5 & 7 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

The problem says $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is in $\text{Nul}A$. Since $\text{Nul}A$ is a subspace of \mathbb{R}^3 , $\begin{bmatrix} 30 \\ 20 \\ -10 \end{bmatrix} = 10 \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, a scalar (10) multiple of $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, must also be in $\text{Nul}A$. That is, $\begin{bmatrix} 30 \\ 20 \\ -10 \end{bmatrix}$ is a solution to the given system.