

SOLUTIONS TO SELECTED PROBLEMS IN SECTION 5.1

10. $\det(A - \lambda I_2) = \det \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix} = 0$ and hence $\lambda = 4$ is an eigenvalue. Now the eigenspace corresponding to 4 is $\text{Nul}(A - \lambda I_2) = \text{Span} \left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$ (How?). Therefore $\left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$ is a basis for the eigenspace (which is one dimensional).

18. The characteristic equation is given by $\begin{vmatrix} 4 - \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 1 & 0 & -3 - \lambda \end{vmatrix} = 0$. Note that the matrix is lower-triangular, so its determinant is just the product of diagonal entries, say $(4 - \lambda)(-\lambda)(-3 - \lambda)$ from which we get $\{4, 0, -3\}$ as solutions.

21. a. False. \vec{x} must be a nonzero vector.

b. True. A is singular if and only if the system $A\vec{x} = \vec{0}$ has a nontrivial solution and this is the same as saying that 0 is an eigenvalue of A .

c. True.

e. False. That's not necessary. You need to find the determinant of the matrix $A - \lambda I_n$ instead.

22. a. False. \vec{x} has to be nontrivial.

b. False. See p.322 for an example.

d. False. If A is a triangular matrix, then true.

e. True. $\text{Nul}(A - \lambda I_n)$, where λ is the corresponding eigenvalue.

23. Eigenvalues of an $n \times n$ matrix A are roots of the characteristic, which is of degree n .

24. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ works. Why?

25. Since λ is an eigenvalue of an invertible matrix A , λ cannot be zero. Therefore, one can find a nontrivial vector \vec{x} which satisfies $A\vec{x} = \lambda\vec{x}$. We claim that the same \vec{x} is an eigenvector of A^{-1} corresponding to eigenvalue $\frac{1}{\lambda}$. Indeed, from $A\vec{x} = \lambda\vec{x}$, we get $\vec{x} = \frac{1}{\lambda}A\vec{x} = A(\frac{1}{\lambda}\vec{x})$. Multiplying both sides by A^{-1} , we obtain $A^{-1}\vec{x} = A^{-1}(A(\frac{1}{\lambda}\vec{x})) = \frac{1}{\lambda}\vec{x}$.

29. The n -vector $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to s . Justify!