SOLUTIONS TO SELECTED PROBLEMS IN SECTION 6.2

2. Orthogonal. In particular, they are linearly independent.

4. Orthogonal. They are, however, <u>not</u> linearly independent. Why doesn't this contradict Theorem 4 in p.384?

6. They are not orthogonal. The inner product of the second and the third vectors is not zero.

10. It is easy to verify that the set is orthogonal. As a consequence, it is linearly independent. Since $\mathbb{R}^3$ is 3-dimensional, by the Basis Theorem in p.259, the vectors in the set form a basis for $\mathbb{R}^3$. To express $\vec{x}$ as a linear combination of those vectors, one needs to solve the system

$$\begin{bmatrix} 3 & 2 & 1 \\ -3 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}.$$

The unique solution of the system is $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ and this means that $\vec{x}$ can be written as $\vec{x} = \frac{4}{3}\vec{u}_1 + \frac{1}{3}\vec{u}_2 + \frac{1}{3}\vec{u}_3$.

14. Let $W = \text{Span}\{\vec{u}\}$. $\vec{y}_W = \text{proj}_W\vec{y}$ is given by $\text{proj}_W\vec{y} = \frac{\vec{y}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u} = \frac{2}{5}\vec{u} = \begin{bmatrix} \frac{14}{5} \\ \frac{2}{5} \end{bmatrix}$. Therefore, $\vec{y} = \begin{bmatrix} \frac{14}{5} \\ \frac{2}{5} \end{bmatrix} + \begin{bmatrix} -\frac{4}{5} \\ \frac{28}{5} \end{bmatrix}$.

15. Let $W = \text{Span}\{\vec{u}\}$. We need to find $\text{dist}(\vec{y}, \vec{y}_W)$. Note that $\vec{y}_W = \begin{bmatrix} \frac{12}{5} \\ \frac{9}{5} \end{bmatrix}$. Finally $\text{dist}(\vec{y}, \vec{y}_W) = \left\| \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix} \right\| = 1$.

26. n orthogonal nonzero vectors are linearly independent in $\mathbb{R}^n$. It follows that these n vectors must be linearly independent by the Basis Theorem and hence they form a basis for $\mathbb{R}^n$.

27. Recall that the components of the product of two matrices are given by inner products of proper rows and columns of those matrices. One can easily show that $UU^T = U^TU = I_n$.

This shows that U is invertible with $U^{-1} = U^T$. Such a matrix (the transpose equals the inverse) is called an *orthogonal* matrix.