SOLUTIONS TO SELECTED PROBLEMS IN SECTION 6.3

4. Let $W = \text{Span}\{\vec{u}_1, \vec{u}_2\}$. It is easy to verify that $\{\vec{u}_1, \vec{u}_2\}$ is orthogonal. $\text{proj}_W \vec{y} = \vec{y}_W = \frac{\vec{y}\cdot\vec{u}_1}{\vec{u}_1\cdot\vec{u}_1}\vec{u}_1 + \frac{\vec{y}\cdot\vec{u}_2}{\vec{u}_2\cdot\vec{u}_2}\vec{u}_2 = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$.

21. **a**. True. Any member $\vec{w}$ in W must be of the form $a\vec{u}_1 + b\vec{u}_2$, where a, b are scalars. Now $\vec{z} \cdot \vec{w} = a\vec{z} \cdot \vec{u}_1 + b\vec{z} \cdot \vec{u}_2 = 0$.
b. True. The Orthogonal Decomposition Theorem.
d. True. Mentioned in class as a remark.

24. **a**. Clear, since the set is pairwise orthogonal.
b. By the Orthogonal Decomposition Theorem, any vector $\vec{y}$ in $\mathbb{R}^n$ can be written as a sum of two vectors, one in W and the other in $W^\perp$, say $\vec{y} = \vec{y}_W + \vec{y}_{W^\perp}$. Now $\vec{y}_W$ can be expressed as a linear combination of $\{\vec{w}_1, \cdots, \vec{w}_p\}$. Similarly, $\vec{y}_{W^\perp}$ can be expressed as a linear combination of $\{\vec{v}_1, \cdots, \vec{v}_q\}$. Therefore, $\vec{y}$ can be expressed as a linear combination of $\{\vec{w}_1, \cdots, \vec{w}_p, \vec{v}_1, \cdots, \vec{v}_q\}$.
c. From part **a** and **b**, we conclude that the set $\{\vec{w}_1, \cdots, \vec{w}_p, \vec{v}_1, \cdots, \vec{v}_q\}$ is a basis for $\mathbb{R}^n$. Since the dimension of $\mathbb{R}^n = n$ and the set contains $p + q$ vectors, we must have $p + q = n$. This establishes the equality in the problem.