SOLUTIONS TO SELECTED PROBLEMS IN SECTION 6.5

4. $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$, $A^TA = \begin{bmatrix} 3 & 3 \\ 3 & 11 \end{bmatrix}$, and $A^T\vec{b} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$. Solving the system $A^TA\vec{x} = A^T\vec{b}$, we get $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as its unique solution. Thus $\hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is (the only) least-squares solution of the given system.

5. $A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, $A^TA = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$, and $A^T\vec{b} = \begin{bmatrix} 14 \\ 4 \\ 10 \end{bmatrix}$. Row reduce the augmented matrix associated with the system $A^TA\vec{x} = A^T\vec{b}$, then we get $\begin{bmatrix} 1 & 0 & 1 & \vdots & 5 \\ 0 & 1 & -1 & \vdots & -3 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$ and this reduced echelon matrix gives us the solution set

$$\begin{cases} x_1 = -x_3 + 5 \\ x_2 = x_3 - 3 \\ x_3 \text{ is free} \end{cases}.$$

17. **a**. True.
b. True
c. False. If you reverse the inequality then True.
d. True.

18. **a**. True. Why do we need the assumption: *if $\vec{b}$ is in the column space of A...* ?
b. True
c. True
d. False. In case A^TA is not invertible (this actually happened in problem **5** above. See problem **25** below also.), $(A^TA)^{-1}$ doesn't make sense. If A^TA is invertible, then the formula in the problem gives a unique least-squares solution to the system $A\vec{x} = \vec{b}$.

25. Solve the least-squares problem $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. The solution set is given by

$$\begin{cases} x = -y + 3 \\ y \text{ is free} \end{cases}.$$