SOLUTIONS TO SELECTED PROBLEMS IN SECTION 1.1

2. The augmented matrix of the given system is

$$\left[\begin{array}{cc:c} 2 & 4 & -4 \\ 5 & 7 & 11 \end{array}\right].$$

Perform suitable row operations to get

$$\left[\begin{array}{cc:c} 2 & 4 & -4 \\ 5 & 7 & 11 \end{array}\right] \xrightarrow{R_1 \mapsto \frac{1}{2}R_1} \left[\begin{array}{cc:c} 1 & 2 & -2 \\ 5 & 7 & 11 \end{array}\right] \xrightarrow{R_2 \mapsto R_2 - 5R_1} \left[\begin{array}{cc:c} 1 & 2 & -2 \\ 0 & -3 & 21 \end{array}\right]$$

$$\xrightarrow{R_2 \mapsto -\frac{1}{3}R_2} \left[\begin{array}{cc:c} 1 & 2 & -2 \\ 0 & 1 & -7 \end{array}\right] \xrightarrow{R_1 \mapsto R_1 - 2R_2} \left[\begin{array}{cc:c} 1 & 0 & 12 \\ 0 & 1 & -7 \end{array}\right].$$

So we have $x_1 = 12$, $x_2 = -7$.

3. Simply solve the system

$$\left[\begin{array}{cc:c} 1 & -2 & -2 \\ 1 & 5 & 7 \end{array}\right],$$

and you will get $x_1 = \frac{4}{7}$, $x_2 = \frac{9}{7}$.

7. The given matrix reduces to $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and the system is inconsistent because the rightmost column is pivotal.

13. Perform suitable row operations to get

$$\left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{R_2 \mapsto R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right]$$

$$\xrightarrow{R_3 \mapsto R_3 - 2R_2} \left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right] \xrightarrow{R_3 \mapsto \frac{1}{5}R_3} \left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow[R_2 \mapsto R_2 - 5R_3]{R_1 \mapsto R_1 + 3R_3} \begin{bmatrix} 1 & 0 & 0 & \vdots & 5 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}.$$

So we have $x_1 = 5$, $x_2 = 3$, $x_3 = -1$.

19. $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \xrightarrow{R_2 \mapsto R_2 - 3R_1} \begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$ and if $6 - 3h = 0$, then the rightmost column would be pivotal and thus the system would be inconsistent. On the other hand, if $6 - 3h \neq 0$, then the second column would be pivotal and the rightmost column cannot be pivotal and thus the system is consistent (with a unique solution). Therefore $h \neq 2$ is the answer.

22. $\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix} \xrightarrow{R_2 \mapsto R_2 + 3R_1} \begin{bmatrix} 2 & -3 & h \\ 0 & 0 & 5+3h \end{bmatrix}$ and in order *not* to get a pivot in the rightmost column, $5 + 3h = 0$ or $h = -\frac{5}{3}$.

24. **a**. True. Mentioned in class as FACTS.
b. False. Two matrices are row equivalent if one of them can be obtained from the other by elementary row operations. See page 7.
c. False. An inconsistent system has no solutions, by definition.
d. True. See the definition in page 3.

29. $R_1 \leftrightarrow R_2$.

32. $R_3 \mapsto R_3 + 3R_2$.