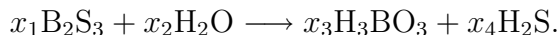


## SOLUTIONS TO SELECTED PROBLEMS IN SECTION 1.6

5. Let's determine the coefficients  $x_1, x_2, x_3, x_4$  in the chemical equation



Comparing the number of  $B$ :  $2x_1 = x_3$ ,

Comparing the number of  $S$ :  $3x_1 = x_4$ ,

Comparing the number of  $H$ :  $2x_2 = 3x_3 + 2x_4$ , and

Comparing the number of  $O$ :  $x_2 = 3x_3$ .

The augmented matrix associated with the system is

$$\left[ \begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right]$$

and it becomes

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

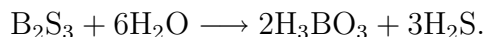
The solution set is now

$$\begin{cases} x_1 = \frac{1}{3}x_4 \\ x_2 = 2x_4 \\ x_3 = \frac{2}{3}x_4 \\ x_4 \text{ is free} \end{cases}.$$

We need one final step. Since each coefficient must be a whole number,  $x_4$  must be a multiple of 3. The simplest is of course 3, so we have

$$\begin{cases} x_1 = 1 \\ x_2 = 6 \\ x_3 = 2 \\ x_4 = 3 \end{cases}$$

and we can balance the chemical equation as follows:



12. (a) Consider the flow-in flow-out equation at each intersection:

At A:  $x_1 = 40 + x_3 + x_4$ ,

At B:  $200 = x_1 + x_2$ ,

At C:  $x_2 + x_3 = 100 + x_5$ , and

At D:  $x_4 + x_5 = 60$ .

The corresponding augmented matrix is

$$\left[ \begin{array}{ccccc|c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array} \right].$$

This matrix row reduces to

$$\left[ \begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

and hence the solution set is

$$\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free} \end{cases}.$$

Since each  $x_i$  is a positive whole number, we have to set some limitations. First of all, in order for  $x_4$  to be nonnegative, we have  $0 \leq x_5 \leq 60$ . Second, both  $x_1$  and  $x_2$  have to be nonnegative and that requires  $-100 \leq x_3 - x_5 \leq 100$ , or equivalently,  $|x_3 - x_5| \leq 100$ .

(b) This time, we have just four variables, say  $x_1, x_2, x_3$ , and  $x_5$ . Again consider the flow-in flow-out equation at each intersection. That will give:

At A:  $x_1 = 40 + x_3$ ,

At B:  $200 = x_1 + x_2$ ,

At C:  $x_2 + x_3 = 100 + x_5$ , and

At D:  $x_5 = 60$ .

The corresponding augmented matrix is

$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 200 \\ 0 & 1 & 1 & -1 & 100 \\ 0 & 0 & 0 & 1 & 60 \end{array} \right],$$

which row reduces to

$$\begin{bmatrix} 1 & 0 & -1 & 0 & \vdots & 40 \\ 0 & 1 & 1 & 0 & \vdots & 160 \\ 0 & 0 & 0 & 1 & \vdots & 60 \\ 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}.$$

The solution set is

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 \text{ is free} \\ x_5 = 60 \end{cases}.$$

Note that we must have  $0 \leq x_3 \leq 160$ , because  $x_2$  should be nonnegative.

(c) If  $x_4 = 0$ , we can regard the road whose flow is  $x_4$  as closed. In part (b), we had  $x_1 = 40 + x_3$  with  $0 \leq x_3 \leq 160$ . Therefore, the minimum value of  $x_1$  is 40. The maximum value of  $x_1$  is 200.