SOLUTIONS TO SELECTED PROBLEMS IN SECTION 2.1

1. $-2A = -2\begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 10 & -4 \end{bmatrix}$.

$B - 2A = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 & 3 \\ -7 & 6 & -7 \end{bmatrix}$.

AC is not defined: A has three columns, while C has two rows.

$CD = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1\cdot 3 + 2\cdot(-1) & 1\cdot 5 + 2\cdot 4 \\ (-2)\cdot 3 + 1\cdot(-1) & (-2)\cdot 5 + 1\cdot 4 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

9. Calculation shows $AB = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$ and $BA = \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$. To get $AB = BA$, we must have (comparing entries in both matrices)

$$-10 + 5k = 15 \qquad \text{and} \qquad -9 = 6 - 3k.$$

Since $k = 5$ is a common solution of these two equations, we conclude that $AB = BA$ if and only if $k = 5$.

10. $AB = AC = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$, but $B \neq C$. This was possible because A is not invertible.

17. First note that B is a 2×3 matrix. Let $\vec{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} c \\ d \end{bmatrix}$ be the first and second columns of B, respectively. The $(1,1)$-entry of AB, -1, is obtained when we inner product the first row of A and the first column of B, so we get $a - 2b = -1$. The $(2,1)$-entry of AB, 6, must be the same as the inner product of the second row of A and the first column of B, giving $-2a + 5b = 6$. Now we have a system with two variables, say a, b, and it's not hard to find the unique solution $a = 7, b = 4$. Similarly, we can set up a system $c - 2d = 2$, $-2c + 5d = -9$ and get $c = -8, d = -5$ as its unique solution. Therefore, we finally have $\vec{v}_1 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$.