SOLUTIONS TO SELECTED PROBLEMS IN SECTION 3.1

2. Using the expansion across the first row,

$$\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix} = 0 \begin{vmatrix} -3 & 0 \\ 4 & 1 \end{vmatrix} - 5 \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & -3 \\ 2 & 4 \end{vmatrix} = 2.$$

4. Let's use the second column this time.

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} = -3 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} - 4 \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = 20.$$

10. Apparently, the second row is the best choice.

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} = -3 \begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix}.$$

Here, using the third row expansion,

$$\begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix} = 5 \begin{vmatrix} -2 & 2 \\ -6 & 5 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix} = 2$$

and hence

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} = (-3) \cdot 2 = -6.$$

You may want to apply row operations to obtain an upper triangular matrix. Of course you get the same result.

12. It is a lower triangular matrix, so the determinant equals the product of diagonal entries. Therefore,

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{vmatrix} = 4 \cdot (-1) \cdot 3 \cdot (-3) = 36.$$

20. The determinant of the second matrix ($k(ad - bc)$) is k times that of the first ($ad - bc$).

22. Both have the same determinant, say $ad - bc$. Note that we used the replacement operation.

38. $\det kA = k^2 \det A = k^2(ad - bc)$. Here the exponent 2 of k comes from the dimension (the number of rows/columns) of A. Generally, if A is an $n \times n$ square matrix, then we have $\det kA = k^n \det A$.