SOLUTIONS TO SELECTED PROBLEMS IN SECTION 3.3

4. $A = \begin{bmatrix} -5 & 3 \\ 3 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$. Therefore $A_1 = \begin{bmatrix} 9 & 3 \\ -5 & -1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} -5 & 9 \\ 3 & -5 \end{bmatrix}$. Now $\det A = -4$, $\det A_1 = 6$, and $\det A_2 = -2$, so $x_1 = \frac{\det A_1}{\det A} = -\frac{3}{2}$ and $x_2 = \frac{\det A_2}{\det A} = \frac{1}{2}$.

6. $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$. Thus $A_1 = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & -2 & 3 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & -2 \end{bmatrix}$. I will leave if for you to check $\det A = 4$, $\det A_1 = -16$, $\det A_2 = 52$, $\det A_3 = -4$. Finally, $x_1 = -4$, $x_2 = 13$, and $x_3 = -1$.

7. The matrix associated with the system is $A = \begin{bmatrix} 6s & 4 \\ 9 & 2s \end{bmatrix}$. For the system to have a unique solution A must be invertible, i.e., $\det A = 12s^2 - 36 \neq 0$. Since $s^2 - 3 = 0$ iff $s = \pm\sqrt{3}$, we have that the system has a unique solution iff $s \neq \pm\sqrt{3}$. Provided $s \neq \pm\sqrt{3}$, we can find the solution using Cramer's rule: $A_1 = \begin{bmatrix} 5 & 4 \\ -2 & 2s \end{bmatrix}$ and $A_2 = \begin{bmatrix} 6s & 5 \\ 9 & -2 \end{bmatrix}$ and hence $\det A_1 = 10s + 8$, $\det A_2 = -12s - 45$. Finally, $x_1 = \frac{10s+8}{12s^2-36} = \frac{5s+4}{6s^2-18}$ and $x_2 = \frac{-12s-45}{12s^2-36} = \frac{-4s-15}{4s^2-12}$.

8. Let $A = \begin{bmatrix} 3s & -5 \\ 9 & 5s \end{bmatrix}$, then $\det A = 15s^2 + 45$ never vanishes, which means whichever s is, A is always invertible. Using Cramer's rule, we have $x_1 = \frac{15s+10}{15s^2+45} = \frac{3s+2}{3s^2+9}$ and $x_2 = \frac{6s-27}{15s^2+45} = \frac{2s-9}{5s^2+15}$.

20. Mark the points in the plane. It is easy to check the parallelogram has $\vec{v}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ as two incident sides. Therefore the area of the parallelogram is $|\det A|$, where $A = \begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix}$. Since $\det A = -7$, the area is 7.

22. Be careful! You cannot apply the determinant method directly. This time the origin is not a vertex of the parallelogram - the fact we learned in class applies only to the parallelogram having the origin as one of its vertices. One possible idea is this. Move one point to the origin and other points accordingly. This translation would not change the area of the parallelogram. For example, if you move $(0, -2)$ to the origin, then the new coordinates of the other three points would be $(-3, 3), (3, 4)$, and $(6, 1)$. Now note that the area is equal to the absolute value of $\begin{vmatrix} -3 & 6 \\ 3 & 1 \end{vmatrix} = -21$, so we get 21.

24. Let's compute $\begin{vmatrix} 1 & -2 & -1 \\ 4 & -5 & 2 \\ 0 & 2 & -1 \end{vmatrix}$. Using the replacement $R_2 \mapsto R_2 - 4R_1$, we have $\begin{vmatrix} 1 & -2 & -1 \\ 4 & -5 & 2 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & -1 \\ 0 & 3 & 6 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 3 & 6 \\ 2 & -1 \end{vmatrix} = -15$. So the volume is the absolute value of the determinant, say 15.