SOLUTIONS TO SELECTED PROBLEMS IN SECTION 4.6

1. Clearly $\text{rank}A = \text{rank}B = 2$ and $\dim \text{Nul}A = 4 - 2 = 2$ by the rank theorem. See the solutions to problems **10** and **16** in **Section 4.3** to find bases for $\text{Nul}A$ and $\text{Col}A$.

2. $\text{rank}A = 3$, $\dim \text{Nul}A = 2$.

3. $\text{rank}A = 3$, $\dim \text{Nul}A = 2$.

4. $\text{rank}A = 3$, $\dim \text{Nul}A = 3$.

6. $\dim \text{Nul}A = 3 - 3 = 0$. Generally, we have that $\text{rank}A^T = \text{rank}A$ (You don't have to memorize this. I didn't mention this in class), so in this example $\text{rank}A^T = 3$.

8. $\dim \text{Nul}A = 6 - 4 = 2$. $\text{Col}A$ is a four-dimensional subspace of $\mathbb{R}^6$, so it is totally different from $\mathbb{R}^4$.

10. $\dim \text{Col}A = \text{rank}A = 6 - 5 = 1$.

16. Since $4 = \text{rank}A + \dim \text{Nul}A$, the smallest possible dimension of $\text{Nul}A$ is achieved when $\text{rank}A$ is maximal. Since $\text{rank}A$ is the number of pivot columns of A, it can be 4 if every column is pivotal. Therefore, the smallest possible dimension of $\text{Nul}A$ is 0. If the matrix is a 4×6 one, then the answer would be 2.