SOLUTIONS TO SELECTED PROBLEMS IN SECTION 5.1

10. $\det(A-\lambda I_2) = \det\begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix} = 0$ and hence $\lambda = 4$ is an eigenvalue. Now the eigenspace corresponding to 4 is $\mathrm{Nul}(A-\lambda I_2) = \mathrm{Span}\left\{\begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}\right\}$ (How?). Therefore $\left\{\begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}\right\}$ is a basis for the eigenspace (which is one dimensional).

18. The characteristic equation is given by $\begin{vmatrix} 4-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 1 & 0 & -3-\lambda \end{vmatrix} = 0$. Note that the matrix is lower-triangular, so its determinant is just the product of diagonal entries, say $(4-\lambda)(-\lambda)(-3-\lambda)$ from which we get $\{4, 0, -3\}$ as solutions.

21. **a**. False. $\vec{x}$ must be a nonzero vector.
b. True. A is singular if and only if the system $A\vec{x} = \vec{0}$ has a nontrivial solution and this is the same as saying that 0 is an eigenvalue of A.
c. True.
e. False. That's not necessary. You need to find the determinant of the matrix $A - \lambda I_n$ instead.

22. **a**. False. $\vec{x}$ has to be nontrivial.
b. False. See p.322 for an example.
d. False. If A is a triangular matrix, then true.
e. True. $\mathrm{Nul}(A - \lambda I_n)$, where λ is the corresponding eigenvalue.

23. Eigenvalues of an $n \times n$ matrix A are roots of the characteristic equation, which is of degree n. According to the fundamental theorem of algebra, this equation has at most n roots.

24. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ works. Why?

25. Since λ is an eigenvalue of an invertible matrix A, λ cannot be zero (See **21** b above). Since λ is an eigenvalue of A, one can find a nontrivial vector $\vec{x}$ satisfying $A\vec{x} = \lambda\vec{x}$. We will prove that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} by showing that this $\vec{x}$ is an eigenvector of A^{-1} corresponding to $\frac{1}{\lambda}$. Indeed, from $A\vec{x} = \lambda\vec{x}$, we get $\vec{x} = \frac{1}{\lambda}A\vec{x} = A(\frac{1}{\lambda}\vec{x})$. Multiplying both sides by A^{-1}, we obtain $A^{-1}\vec{x} = A^{-1}(A(\frac{1}{\lambda}\vec{x})) = \frac{1}{\lambda}\vec{x}$.

29. The n-vector $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to s. Justify!