SOLUTIONS TO SELECTED PROBLEMS IN SECTION 6.1

1. $\vec{u} \cdot \vec{u} = 5$, $\vec{v} \cdot \vec{u} = 8$, $\frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} = \frac{8}{5}$.

4. $\frac{1}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{1}{5} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} -\frac{1}{5} \\ \frac{2}{5} \end{bmatrix}$.

7. $\|\vec{w}\| = \sqrt{9+1+25} = \sqrt{35}$.

10. Note that $\left\| \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix} \right\| = \sqrt{36+16+9} = \sqrt{61}$, therefore, the unit vector having the same direction as the original vector is $\frac{1}{\sqrt{61}} \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$ or $\begin{bmatrix} -\frac{6}{\sqrt{61}} \\ \frac{4}{\sqrt{61}} \\ -\frac{3}{\sqrt{61}} \end{bmatrix}$.

14. $\operatorname{dist}(\vec{u}, \vec{z}) = \|\vec{u} - \vec{z}\| = \left\| \begin{bmatrix} 4 \\ -4 \\ -6 \end{bmatrix} \right\| = \sqrt{16+16+36} = \sqrt{68} = 2\sqrt{17}$.

16. $\vec{u} \cdot \vec{v} = 24 - 9 - 15 = 0$, so $\vec{u}$ and $\vec{v}$ are orthogonal.

24. Note that in general $\|x\|^2 = \vec{x} \cdot \vec{x}$ for any $\vec{x} \in \mathbb{R}^n$. Now we have

$$\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v} = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u} \cdot \vec{v}.$$

Similarly,

$$\|\vec{u} - \vec{v}\|^2 = (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \vec{u} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v} = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\vec{u} \cdot \vec{v}.$$

Add two equalities above to get the result.

27. By assumption, $\vec{y} \cdot \vec{u} = 0$ and $\vec{y} \cdot \vec{v} = 0$. We want to show that $\vec{y} \cdot (\vec{u} + \vec{v}) = 0$. Indeed, $\vec{y} \cdot (\vec{u} + \vec{v}) = \vec{y} \cdot \vec{u} + \vec{y} \cdot \vec{v} = 0 + 0 = 0$, as desired.

28. By assumption, $\vec{y} \cdot \vec{u} = 0$ and $\vec{y} \cdot \vec{v} = 0$. Choose $\vec{w} \in \operatorname{Span}\{\vec{u}, \vec{v}\}$. By definition of $\operatorname{Span}\{\vec{u}, \vec{v}\}$, $\vec{w}$ must be of the form $\vec{w} = c_1\vec{u} + c_2\vec{v}$ for suitable scalars c_1 and c_2. Now $\vec{y} \cdot \vec{w} = \vec{y} \cdot (c_1\vec{u} + c_2\vec{v}) = \vec{y} \cdot (c_1\vec{u}) + \vec{y} \cdot (c_2\vec{v}) = c_1\vec{y} \cdot \vec{u} + c_2\vec{y} \cdot \vec{v} = 0$.

Remark: This problems shows that to show that $\vec{y}$ is in $V^{\perp}$, it is enough to show that

$\vec{y} \cdot \vec{v}_i = 0$ for each i, $1 \leq i \leq p$, where $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$ is a basis for V.

31. Note that $\vec{x} \cdot \vec{x} = 0$ (Why?). Now $\|\vec{x}\| = \vec{x} \cdot \vec{x} = 0$. Since $\vec{0}$ is the only vector having zero length, it follows that $\vec{x} = \vec{0}$.