Homework 8

1) a) Need $\mathrm{Nul}\begin{pmatrix} -6 & -2 \\ -3 & -1 \end{pmatrix}$. basis is $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

So $E_{10} = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\}$

b) Need $\mathrm{Nul}\begin{pmatrix} -1 & 0 & 2 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} -1 & 0 & 2 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 2 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ basis is $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \\ 0 \end{pmatrix} \right\}$

$E_4 = \mathrm{Span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \\ 0 \end{pmatrix} \right\}$

2) a) $\det\begin{pmatrix} 2-\lambda & 7 \\ 7 & 2-\lambda \end{pmatrix} = (2-\lambda)^2 - 49$

zero when $(2-\lambda)^2 = 49$, i.e. $2-\lambda = \pm 7$

$\lambda = 9, -5$

b) $\det\begin{pmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{pmatrix} = (2-\lambda)\det\begin{pmatrix} -1-\lambda & 0 \\ -3 & 4-\lambda \end{pmatrix}$

$= (2-\lambda)(-1-\lambda)(4-\lambda)$

eigenvalues $2, 4, -1$

3) $\det(A-\lambda I) = (5-\lambda)(3-\lambda)(5-\lambda)(1-\lambda)$

So the algebraic multiplicity of $\lambda=5$ is 2.

We need the geometric multiplicity ($=\dim \mathrm{Nul}(A-5I)$) to be 2, i.e. we need

$$\operatorname{rank}\begin{pmatrix} 0 & -2 & 6 & -1 \\ 0 & -2 & h & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -5 \end{pmatrix} = 2.$$

this will happen when there are pivots in columns 2, 4 i.e. when $h=6$.

4) $\lambda=5$: $E_5 = \mathrm{Span}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$

$\lambda=1$ $E_1 = \mathrm{Span}\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\-1\\0\end{pmatrix}\right\}$