

Homework 9

$$1) \text{ distance } \left(\begin{pmatrix} 10 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \end{pmatrix} \right) = \left\| \begin{pmatrix} 11 \\ -8 \end{pmatrix} \right\|$$

$$= \sqrt{121 + 64} = \sqrt{185}$$

2) a) true by definition of length.

b) true: if $\|\vec{u} - \vec{v}\| = \|\vec{u} + \vec{v}\|$ then

$$(\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$$

$$\vec{u} \cdot \vec{u} - \vec{v} \cdot \vec{u} - \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} \stackrel{\text{II}}{=} \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}$$

$$-2\vec{u} \cdot \vec{v} \stackrel{\text{II}}{=} 2\vec{u} \cdot \vec{v}$$

$$\vec{u} \cdot \vec{v} \stackrel{\text{II}}{=} 0 \Leftrightarrow \vec{u} \text{ and } \vec{v} \text{ are perpendicular}$$

$$3) A = \begin{pmatrix} 1 & -2 & -3 \end{pmatrix}$$

$$4) \vec{x} = \frac{\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}}{\begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}} \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} + \frac{\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}}{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \frac{\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$$= \frac{24}{18} \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} + \frac{3}{9} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \frac{6}{18} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$$5) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\begin{pmatrix} -1 \\ 3 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix}} \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}^{\perp} = \frac{-4}{10} \begin{pmatrix} -1 \\ 3 \end{pmatrix} + v^{\perp}$$

$$\Rightarrow \frac{-2}{5} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$6) \quad y = \frac{\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \frac{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}}{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \vec{v}^\perp$$

$$= \frac{3}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \vec{v}^\perp$$

$$= \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} + \vec{v}^\perp$$

$$\text{answer} = \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix}$$