Exam 1

NAME:

November 29, 2008

Question 1. (8 points) Compute the following products of matrices.

a) $\begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -9 \\ 8 \end{pmatrix}$

b) $\begin{pmatrix} 4 & 5 & 1 \\ 3 & 7 & 1 \end{pmatrix} \begin{pmatrix} -1/3 & 1 \\ 1/3 & 1 \\ -1/3 & -9 \end{pmatrix}$

Question 2. (6 points) Find the inverse of the following matrix.

$$\begin{pmatrix} 1 & -4 & 3 \\ 1 & -3 & 5 \\ -2 & 7 & -7 \end{pmatrix}$$

Question 3. (12 points) True or False? For each true statement, provide a brief explanation why it's true. For each false statement, give a specific counterexample.

a) The zero vector is a solution to every consistent linear system.

b) Every linear system has the same number of solutions as its associated homogeneous system.

c) If $\{v_1, v_2, v_3, v_4\}$ are linearly *independent*, then $\{v_1, v_3, v_4\}$ are also linearly independent.

d) If the columns of a square matrix A span $\mathbb{R}^n$, then the equation $A^2\vec{x} = \vec{0}$ has at least 7 solutions.

Question 4. (9 points) Give an example of:

a) A matrix with linearly independent columns that *cannot* be row reduced to the identity matrix.

b) A square matrix which does not have any zero entries and is not invertible.

c) A pair of 2×2 matrices that commute with each other.

Question 5. (6 points) In this question, A, B, X are all square matrices of the same size, and let

$$(A - AX)^{-1} = X^{-1}B$$

(When $^{-1}$ is added to a matrix, it means the inverse exists.) Explain why B is invertible.

Question 6. (4 points) Let C be a 4×4 matrix, and let the two first columns of C be

$$\begin{pmatrix} 2 \\ 1 \\ 3 \\ -4 \end{pmatrix} \begin{pmatrix} -4 \\ -2 \\ -3 \\ 8 \end{pmatrix}$$

(You don't know anything about the two other columns of C.) Find three different solutions to $C\vec{x} = \vec{0}$.

Question 7. (12 points) Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 8 \\ 3 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 10 \\ 2 \end{pmatrix}$.

a) Show that $\vec{b}$ can be written as a linear combination of $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ by finding a set of weights of that linear combination.

b) Are there any vectors in $\mathbb{R}^3$ that cannot be written as a linear combination of $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$?

Question 8. (9 points) Find *all* values for h, k such that the system below is:

$$\begin{aligned} x_1 + 5x_2 &= 3 \\ -3x_1 + hx_2 &= k \end{aligned}$$

a) underdetermined.

b) uniquely determined.

c) overdetermined.

Question 9. (9 points) Determine whether the following matrices are invertible. *Do not* calculate inverses. Explain your answer.

a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 3 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & -2 & 8 \end{pmatrix}$