Some Solutions to practice problems.

Exercise 6

the columns of B are not linearly independent, so there is a nonzero vector $\overrightarrow{frankie}$ such that

$$B\left(\overrightarrow{frankie}\right) = \vec{0}$$

then Also

$$AB\,\overrightarrow{frankie} = A\vec{0} = \vec{0}$$

So $\overrightarrow{frankie}$ is a solution with some nonzero entries of

$$(AB)\vec{x} = \vec{0}$$

So the columns of (AB) are linearly dependent.

Exercise 7

1) D is $n \times 3$

2) the first column of D, $\vec{d}_1$ is a solution to

$$A\vec{d}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{since}$$

$$AD = A\begin{pmatrix} \vec{d}_1 & \vec{d}_2 & \vec{d}_3 \end{pmatrix} = \begin{pmatrix} A\vec{d}_1 & A\vec{d}_2 & A\vec{d}_3 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

the second column of D solves

$$A\vec{d}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

etc.

Question 15

$$B = \left(\vec{b}_1 \ \ \vec{b}_2 \ \ \vec{b}_3 \ \ \dots \right)$$

$$AB = \left(A\vec{b}_1 \ \ A\vec{b}_1 \ \ A\vec{b}_3 \ \dots \right)$$

So the first two columns of A are equal.

Question 17

We need a vector that solves $A\vec{x} = \vec{b}$

We know $AD = I$

To get a $\vec{b}$ in this equation, multiply by $\vec{b}$ from the right.

$$AD\vec{b} = I\vec{b} = \vec{b}$$

So $A(D\vec{b}) = \vec{b}$ and $D\vec{b}$ is a solution.

Question 20

As there is a $\vec{z}$ such that $H\vec{x} = \vec{z}$ does not have a solution, so H is not invertible (IMT)

Question 22

AB is invertible, call it C

C^{-1} exists

$$ABC^{-1} = (AB)C^{-1} = CC^{-1} = I$$

so $A\left(BC^{-1}\right) = I$ and A is square.

so the inverse of A is $\left(BC^{-1}\right)$