Solutions Homework 4

1)

$$\begin{pmatrix} 2 & 1 \\ 3 & 1 \\ -1 & -1 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -1 & 2 & 0 \\ 1 & -2 & -1 & 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & -3 & 7 & -1 \\ 7 & 1 & -4 & 9 & -1 \\ -3 & 1 & 2 & -5 & 1 \\ 9 & 2 & 5 & 11 & -1 \end{pmatrix}$$

2) for the first column, $A\vec{b_1} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

for the second column, $A\vec{b_2} = \begin{pmatrix} 2 \\ -9 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 1 & -2 & \cancel{-6}\ -1 & 2 \\ -2 & 5 & 6 & -9 \end{array}\right)$$

$$\xrightarrow{R2+2R1} \left(\begin{array}{cc|cc} 1 & -2 & -1 & 2 \\ 0 & 1 & 4 & -5 \end{array}\right)$$

$$\xrightarrow{R1+2R2} \left(\begin{array}{cc|cc} 1 & 0 & 7 & -8 \\ 0 & 1 & 4 & -5 \end{array}\right)$$

$$\vec{b_1} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} \quad \text{and} \quad \vec{b_2} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$$

3) a) $A = CB^{-1} = ABB^{-1} = A$

C and B^{-1} are invertible, A is the product of 2 invertible matrices.

b) the columns of D are linearly independent
$\Rightarrow D$ is invertible (IMT)

$\Rightarrow D^{-1}$ exists

$$D(B-C) = 0$$

$$D^{-1}D(B-C) = 0$$

$$B = C$$

c) A^{-1} is invertible: $(A^{-1})^{-1}$ is A
A^{-1} has linearly independent columns

d) $A^2 = A \cdot A$ so A^2 is the product of invertible matrices, A^2 is invertible.
A^2 has columns that span $\mathbb{R}^n$ (IMT)

4) $AB = \begin{pmatrix} -7 & -10+5k \\ -9 & 15+k \end{pmatrix}$

$BA = \begin{pmatrix} -7 & 15 \\ 6-3k & 15+k \end{pmatrix}$ $k=-1$ makes these equal.

5)

$$\vec{x} = A^{-1}A\vec{x} = A^{-1}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 6 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -10 \\ -7 \end{pmatrix}$$

6) $\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 3 & 1 & -2 & 0 & 1 & 0 \\ -5 & -1 & 7 & 0 & 0 & 1 \end{array}\right)$

$\begin{array}{l} R2-3R1 \\ R3+5R1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & -1 & -3 & 5 & 0 & 1 \end{array}\right)$

$R3+R2 \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right)$

$\begin{array}{l} R1+2R3 \\ R2-4R3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & +2 & +2 \\ 0 & 1 & 0 & -8 & -3 & -4 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right)$ $\begin{pmatrix} 5 & 2 & 2 \\ -8 & -3 & -4 \\ 2 & 1 & 1 \end{pmatrix}$

7)

$$\left(\begin{array}{ccc|cc} 2 & 1 & 7 & 1 & +1 \\ 3 & 1 & -1 & 2 & -1 \end{array}\right)$$

$$R2-\frac{3}{2}R1 \quad \left(\begin{array}{ccc|cc} 2 & 1 & 7 & 1 & 1 \\ 0 & -1/2 & \frac{-23}{2} & 1/2 & -5/2 \end{array}\right)$$

$$2R2 \quad \left(\begin{array}{ccc|cc} 2 & 1 & 7 & 1 & 1 \\ 0 & 1 & 23 & -1 & 5 \end{array}\right)$$

$$R1-R2 \quad \left(\begin{array}{ccc|cc} 2 & 0 & -16 & 2 & -4 \\ 0 & 1 & 23 & -1 & 5 \end{array}\right)$$

$$\tfrac{1}{2}R1 \quad \left(\begin{array}{ccc|cc} 1 & 0 & -8 & 1 & -2 \\ 0 & 1 & 23 & -1 & 5 \end{array}\right)$$

first system: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ -23 \\ 1 \end{pmatrix}$

Second system $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ -23 \\ 1 \end{pmatrix}$