Math 213, Section B1, Quiz 2 (Solutions); Friday, Jan 25, 2008

PRINT YOUR NAME:

1.
Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = e^x - 1$.
(1) Is the function f one-to-one?
(2) Find the range of f.
(3) Is f surjective?
(4) For $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2 + x$, compute the functions $f \circ g$ and $g \circ f$.

Solution.

(1) The function $f(x)$ is one-to-one. Indeed, if $f(x_1) = f(x_2)$ for some $x_1 = x^2 \in \mathbb{R}$ then $e^{x_1} - 1 = e^{x_2} - 1$. Hence $e^{x_1} = e^{x_2}$ and $x_1 = x_2$.

(2) The range of f is $(-1, \infty) = \{y \in \mathbb{R} \,|\, y > -1\}$.
Indeed, by definition,

$$\text{range}(f) = \{f(x)|x \in \mathbb{R}\} = \{e^x - 1 \,|\, x \in \mathbb{R}\} =$$
$$\{y \in \mathbb{R} \,|\, y = e^x - 1 \text{ for some } x \in \mathbb{R}\} = \{y \in \mathbb{R} \,|\, y - 1 = e^x \text{ for some } x \in \mathbb{R}\} =$$
$$= \{y \in \mathbb{R} \,|\, y - 1 > 0\} = \{y \in \mathbb{R} \,|\, y > -1\}.$$

(3) No, the function f is not surjective. For example, the number -3 belongs to the co-domain of f (which, by definition of f, is the set $\mathbb{R}$), but $-3 \notin (-1, \infty) = \text{range}(f)$.

(4) We have

$$(f \circ g)(x) = f(g(x)) = f(x^2 + x) = e^{x^2+x} - 1,$$

$$(g \circ f)(x) = g(f(x)) = g(e^x - 1) = (e^x - 1)^2 + e^x - 1 = e^{2x} - e^x.$$