Math 213, Quiz 7 (Solution); Friday, March 7, 2008

1.
(a) Find the general solution of the following recurrence relation:

$$a_n = -6a_{n-1} - 9a_{n-2}.$$

(b) Find a sequence a_n such that

$$\begin{cases} a_n = -6a_{n-1} - 9a_{n-2}, & n \geq 2 \\ a_0 = 1, a_1 = 0. \end{cases}$$

Provide a detailed explanation of your answers.

Solution.
(a) The characteristic equation of this recurrence relation is:

$$r^2 = - - 6r - 9$$
$$r^2 + 6r + 9 = 0$$
$$(r+3)^2 = 0.$$

Hence the general solution of the recurrence relation $a_n = -6a_{n-1} - 9a_{n-2}$ is:

$$a_n = (\alpha + \beta n)(-3)^n = \alpha(-3)^n + \beta n(-3)^n,$$

where $\alpha, \beta \in \mathbb{R}$ are arbitrary constants.

(b) Using the result of (a), we know that a_n has the form $a_n = \alpha(-3)^n + \beta n(-3)^n$ for some $\alpha, \beta \in \mathbb{R}$. To find the specific values of α and β, we use the initial conditions $a_0 = 1, a_1 = 0$ that yield:

$$\begin{cases} \alpha(-3)^0 + \beta \; 0 \; (-3)^0 = 1 \\ \alpha(-3)^1 + \beta n(-3)^1 = 0 \end{cases} \Rightarrow \begin{cases} \alpha = 1 \\ -3\alpha - 3\beta = 0, \end{cases} \Rightarrow \begin{cases} \alpha = 1 \\ \beta = -1. \end{cases}$$

Thus $a_n = (-3)^n - n(-3)^n$.