H/Wk 8

4.6 1. (a) $X = \sin\theta$

$dx = \cos\theta d\theta$

$$\int_0^1 (1-x^2)^{3/2} dx = \int_0^{\pi/2} (1-\sin^2\theta)^{3/2} \cos\theta d\theta$$

$$= \int_0^{\pi/2} \cos^4\theta d\theta$$

$$= \int_0^{\pi/2} \cos^2\theta \cos^2\theta d\theta$$

$$= \int_0^{\pi/2} \frac{1+\cos 2\theta}{2} \cdot \frac{1+\cos 2\theta}{2} d\theta$$

$$= \int_0^{\frac{\pi}{2}} 1+2\cos\theta + \cos^2 2\theta$$

$$= \int_0^{\pi/2} \left[\frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2 2\theta\right] d\theta$$

$$= \int_0^{\pi/2} \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{4} \frac{1+\cos 4\theta}{2} d\theta$$

$$= \frac{3}{16}\pi$$

(b) $X = U^2-1$ $\quad dx = 2u du$

$$\int_0^1 \frac{1}{1+\sqrt{1+x}} dx = \int_1^{\sqrt{2}} \frac{1}{1+u} 2u du = 2\int_1^{\sqrt{2}} \frac{u}{1+u} du = 2\left[1+u - \ln|1+u|\right]_1^{\sqrt{2}}$$

$$= 2\sqrt{2} - 2 + 2\log(2\sqrt{2}-2)$$

(c) $t = \tan\left(\frac{x}{2}\right)$ $\quad \tan^{-1}(t) = \frac{x}{2}$ $\quad dx = \frac{2}{1+t^2} dt$

$$\int_0^{\pi/2} \frac{1}{\sin x + \cos x + 2} dx = \int_0^1 \frac{1}{\frac{2t + 1-t^2}{1+t^2} + 2} \cdot \frac{2}{1+t^2} dt$$

$$= \int_0^1 \frac{2}{t^2+2t+3} dt = \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x+\sqrt{2}}{2}\right)\Big|_0^1$$

$$= 2\tan^{-1}(\sqrt{2}) - \sqrt{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

(d) $t = 1 + x\cos x \qquad dt = (-x\sin x + \cos x)\,dx$

$$\int_0^{\pi/4} \frac{x\cos x\,(x\sin x - \cos x)}{1 + x\cos x}\,dx = \int_1^{1+\frac{\pi\sqrt{2}}{8}} \frac{(t-1)(-dt)}{t} = \int_1^{1+\frac{\sqrt{2}\pi}{8}} \left(\frac{1}{t} - 1\right)dt$$

$$= \ln t - t \Big|_1^{1+\frac{\sqrt{2}\pi}{8}} = \log\left(\frac{\sqrt{2}\pi}{8}\right) + 1 - \frac{\sqrt{2}\pi}{8}$$

17. (a) $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$

$$\int_0^1\int_0^x \ln(1+x^2+y^2)\,dy\,dx = \int_0^{1/2}\int_v^{1-v} \ln(1+(u+v)^2+(u-v)^2)\,|-2|\,du\,dv$$

$$= 2\int_0^{1/2}\int_v^{1-v} \ln(1+u^2+2uv+v^2+u^2-2uv+v^2)\,du\,dv$$

$$= 2\int_0^{1/2}\int_v^{1-v} \ln(1+2u^2+2v^2)\,du\,dv$$

(b) $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$$\int_0^1\int_{1-x}^{1+x} \sqrt{1+x^2y^2}\,dy\,dx = \iint \sqrt{1+u^2(u+v)^2}\,|1|\,du\,dv$$

$$= \iint \sqrt{1+u^2(u^2+2uv+v^2}\,du\,dv$$

$$= \iint -\sqrt{1+u^4+2u^3v+u^2v^2}\,dv\,du$$

$$= \int_0^1\int_{1-2u}^{1} \sqrt{1+u^2(u+v)^2}\,dv\,du$$

4.7 (1) $x = a\cos\theta \quad y = a\sin\theta$

$$S = \int_0^{2\pi} \sqrt{(-a\sin\theta)^2 + (a\cos\theta)^2}\, d\theta = \int_0^{2\pi} \sqrt{a^2(\sin^2\theta + \cos^2\theta)}\, d\theta = \int_0^{2\pi} a\, d\theta = 2\pi.$$

(b) $x = a\frac{1-t^2}{1+t^2} \quad y = a\frac{2t}{1+t^2} \Rightarrow \frac{dx}{dt} = \frac{-4at}{(t^2+1)^2} \quad \frac{dy}{dt} = \frac{-2a(t^2-1)}{(t^2+1)^2}$

$$\Rightarrow \int_{-\infty}^{\infty} \sqrt{\frac{4a^2}{(t^2+1)^2}} = 2\pi a.$$

6.(a) $z = f(x) \quad y = 0$

let $r = x_1 \quad z = z_1$

For any fixed point (r, z) in cylinderical, $0 \le \theta \le 2\pi$, $r = x_1$, $z = \cdot$

So we have a circle, $z = f(r)$

(b) $S = \iint_{R_{r\theta}} \sqrt{EG - F^2}\, dr\, d\theta = \int_0^{2\pi} \int_a^b \sqrt{(1 + f'(r)^2) - 0^2}\, r\, dr\, d\theta$

$$\left(\begin{array}{ll} \theta = \theta & 0 \le \theta \le 2\pi \\ r = r & a \le r \le b. \\ z = f(r) & \end{array} \right.$$

$$= \int_0^{2\pi} \int_a^b \sqrt{1 + f'(r)^2}\, r\, dr\, d\theta$$

$$= 2\pi \int_a^b \sqrt{1 + f'(r)^2}\, r\, dr$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = r$$

4.9 1. (a) $F = \int_{\pi/2}^{\pi} \frac{\cos(xt)}{x} dx$ $\quad \frac{dF}{dt} = \int_{\pi/2}^{\pi} x \cdot \frac{-\sin(xt)}{x} dx = -\int_{\pi/2}^{\pi} \sin(xt)\, dx$

(b) $F = \int_1^2 \frac{x^2}{(1-tx)^2} dx \Rightarrow \frac{dF}{dt} = \int_1^2 -2x^2(1-tx)^{-3}(-x)\, dx = \int_1^2 \frac{2x^3}{(1-tx}$

(c) $F = \int_1^2 \ln(xu)dx$ $\quad \frac{dF}{du} = \int_1^2 \frac{x}{xu} dx = \int_1^2 \frac{1}{u} dx$

(d) $F = \int_1^2 \frac{\sin x}{x-y} dx$ $\quad \frac{d^nF}{dy^n} = \int_1^2 \sin x\,(x-y)^{-1-n} \cdot n\,(n-1) \cdots dx$

$$= n! \int_1^2 \frac{\sin x}{(x-y)^{n+1}} dx$$

2 (a) $F = \int_1^x t^2 dx \Rightarrow \frac{dF}{dx} = \int_1^x 0\,dt + x^2 \cdot 1 + 1^2 \cdot 0 = x^2$

(b) $F = \int_1^{t^2} \sin(x^2)dx \Rightarrow \frac{dF}{dt} = \int_1^{t^2} 0\,dx + \sin(t^4) \cdot 2t - \sin(1) \cdot 0 = 2t\sin(t^4)$

(c) $F = \int_{t^3}^{2} \ln(1+x^2)\, dx \Rightarrow \frac{dF}{dt} = \int_{t^3}^{2} 0\,dx + \ln(1+2^2) \cdot 0 - \ln(1+t^6) \cdot 3t^2$

$$= -3t^2 \ln(1+t^6)$$

(d) $F = \int_x^{\tan x} e^{-t^2} dt \Rightarrow \frac{dF}{dx} \int_x^{\tan x} 0\,dt + e^{-\tan^2 x}\sec^2 x - e^{-x^2} \cdot 1$

$$= \sec^2 x\, e^{-\tan^2 x} - e^{-x^2}$$