SAMPLE MULTIPLE CHOICE QUESTIONS FOR THE 2-ND MIDTERM.

The second midterm will take place on Monday, April 9. It will cover the material of the following sections: Ch. 3.1-3.6; Ch. 4.3-4.7, Ch. 4.9; Ch 5.1-5.6. The exam will be structured exactly as the first midterm: there will be one multiple choice theoretical problem with several parts and four computational problems. Review problems are homework problems assigned for these chapters. You should expect the computational problems on the exam to be somewhat easier than homework problems and somewhat harder than the quiz problems. As before, you are allowed to use one two-sided sheet of notes on the test.

Choose the correct answer among the options provided.

(a) Let $v(x, y, z)$ be a continuous vector field in some domain D in $\mathbb{E}^3$ such that the components v_x, v_y, v_z of v have continuous first-order partial derivatives in D.

(1) If $\operatorname{curl} v = 0$ in D then $v = \operatorname{grad} f$ for some function $f(x, y, z)$ in D.
(2) We always have $\operatorname{div} \operatorname{curl} v = 0$ in D.
(3) If $v = \operatorname{grad} f$ for some function $f(x, y, z)$ in D with continuous partial derivatives of order two in D then $\operatorname{curl} v = 0$.
(4) None of the above

Answer: (3)

(b) Let $u(x, y, z)$ be a vector field such that $u(0, 0, 0) = [1, 0, 1]$ and $\operatorname{div} u|_{(0,0,0)} = 3$. Then

$$\operatorname{div} \left[(x^2 + 2x + y^2 - z^2 + 5)u\right]|_{(0,0,0)}$$

is equal to

Then:

(1) 0
(2) 17.
(3) 15.
(4) None of the above

Answer: (2)

(c) The limit

$$\lim_{t \to 0} \frac{1}{\pi t^2} \int_{x^2+y^2 \leq t^2} e^{x^3+y^3+1} \cos(x^2 y^2)\, dx\, dy$$

is equal to

(1) 1;
(2) 0;
(3) e;
(4) does not exist.

Answer: (3)

(d) Let R_{xy} be a region in the xy-plane such that the change of variables $x = x(u, v), y = y(u, v)$ provides a differentiable one-to-one map from a region R_{uv} in the uv-plane to R_{xy}. Suppose the area of R_{uv} is 20 and that $\frac{\partial(x,y)}{\partial(u,v)} = -2$ for any (u, v) in R_{uv}.

Then the area of R_{xy} is equal to

(1) 10;
(2) -40;
(3) 40;
(4) None of the above.

Answer: (3)

(e) Let R_{xy} be the closed region bounded by a circle of radius 2 centered at $(5,4)$ in the xy-plane. Let $f(x,y)$ be a continuous function on $R_{x,y}$.

Then the integral $\iint_{R_{xy}} f(x,y)dx\,dy$:

(1) is equal to
$$\int_3^7 [\int_{4-\sqrt{10x-x^2-21}}^{4+\sqrt{10x-x^2-21}} f(x,y)\,dx]\,dy;$$
(2) does not always exist;
(3) is equal to
$$\int_3^7 [\int_{4-\sqrt{10x-x^2-21}}^{4+\sqrt{10x-x^2-21}} f(x,y)\,dy]\,dx;$$
(4) None of the above

Answer: (3)

(f) Let S be a surface in the xyz-space. Suppose S is parameterized by a continuous and differentiable map from the region $u^2+v^2 \leq 5$ in the uv-plane given by $x = x(u,v), y = y(u,v), z = z(u,v)$ and such that the Jacobi matrix of this map is equal to $\begin{bmatrix} 3 & 0 \\ 1 & -1 \\ 0 & 2 \end{bmatrix}$ for all (u,v).

Then the area of S is equal to:

(1) $5\pi\sqrt{14}$;
(2) $25\pi\sqrt{14}$;
(3) 70π
(4) None of the above

Answer: (4)

(g) Let $f(x,y)$ be a function which is continuous and has continuous first partial derivatives everywhere in the xy-plane. Suppose $f(y^3,y) = \sin y$ for any y. Then
$$\frac{d}{dy}\int_0^{y^3} f(x,y)\,dx$$
is equal to:

(1) $\int_0^{y^3} \frac{\partial f}{\partial y}(x,y)\,dx$;
(2) $-3y^2 \sin y + \int_0^{y^3} \frac{\partial f}{\partial y}(x,y)\,dx$;
(3) $3y^2 \sin y + \int_0^{y^3} \frac{\partial f}{\partial y}(x,y)\,dx$;
(4) None of the above

Answer: (3)

(h) Let $f(x,y)$ be a function which is continuous everywhere in the xy-plane and such that $-5 \leq f(x,y) \leq 3$ for any (x,y). Let C be a smooth curve in the xy-plane of length 10.

Then
(1) $\int_C f(x,y)\,ds \le 30$;
(2) $\int_C f(x,y)\,ds \le 50$;
(3) $\int_C f(x,y)\,ds \ge 0$;
(4) None of the above

Answer: (2)

(i) Consider the vector field $\mathbf{v}(x,y) = 2x\mathbf{i} + y\mathbf{j}$ in the xy-plane. Let C be a smooth simple closed curve in the xy-plane traveled counterclockwise and enclosing the region of area 10. Let $\mathbf{n}$ denote the outer unit normal vector to C.

Then

$$\oint_C \mathbf{v}\cdot\mathbf{n}\,ds$$

is equal to
(1) 0
(2) 3;
(3) 10;
(4) None of the above

Answer: (4) [The integral is equal to 30.]

(j) Let $F(x,y)$ be a function in the xy-plane which is continuous and has continuous partial derivatives everywhere in the xy-plane. Let C be the curve $x = t + t^3, y = t^4 - 2t$, $0 \le t \le 1$. Suppose $F(0,0) = 5$ and $F(2,-1) = 13$.

Then

$$\int_C \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dx$$

is equal to
(1) -8
(2) 0;
(3) 8;
(4) None of the above

Answer: (3)