Math 280 Section C1 Quiz 7

March 23, 2001

Problem 1.

Compute the following line integral:

$$\oint_{x^2+y^2=4} \sqrt{x^2+y^2}\, dx.$$

Solution.

The curve $x^2 + y^2 = 4 = 2^2$ is a circle of radius 2. We will parameterize it in the counter-clockwise direction as

$$\begin{cases} x = 2\cos\theta \\ y = 2\sin\theta \end{cases} \text{ where } 0 \le \theta \le 2\pi.$$

Then

$$\oint_{x^2+y^2=4} \sqrt{x^2+y^2}\, dx = \int_0^{2\pi} \sqrt{4\cos^2\theta + 4\sin^2\theta}\, (-2\sin\theta)\, d\theta =$$

$$= -4\int_0^{2\pi} \sin\theta\, d\theta = 4[\cos\theta]_0^{2\pi} = 0.$$