Quiz 1; September 15 (with Solutions)

(1) Indicate which choice below best describes the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + e^x y = 0$$

(a) The equation is linear.

(b) The equation is separable.

(c) The equation is both linear and separable.

(d) The equation is neither linear nor separable.

Answer: (c)

(2) Consider the initial value problem

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{\cos(xy)}, \\ y(0) = 1. \end{cases}$$

For each of the statements below indicate if it does or does not follow from Theorem 1 (Existence and Uniqueness of Solutions) in Ch. 1.3:

(a) This initial value problem has at least one solution on the interval $-1 < x < 1$. [Answer: Does not follow]

(b) This initial value problem has a unique solution on the interval $-1 < x < 1$. [Answer: Does not follow]

(c) This initial value problem does not have a solution on the interval $-1 < x < 1$. [Answer: Does not follow]

(d) This initial value problem has a unique solution on the interval $-\epsilon < x < \epsilon$ for some $\epsilon > 0$. [Answer: Follows]

(3) Find the general solution of the equation $y' + 3y^2 = 0$ on the interval $x > 0$.
Solution. This is a separable equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 3y^2 = 0, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -3y^2$$
$$-\frac{\mathrm{d}y}{y^2} = 3\,\mathrm{d}x, \quad \int -\frac{\mathrm{d}y}{y^2} = \int 3\,\mathrm{d}x$$
$$\frac{1}{y} = 3x + C, \quad y = \frac{1}{3x + C}.$$

In the process of separating the variables we had to divide the equation by y^2, which could have produced a singular solution. Indeed, $y^2 = 0$ gives $y = 0$. By direct substitution we see that $y = 0$ is in fact a solution of the original equation $y' + 3y^2 = 0$.

Thus the general solution of the equation $y' + 3y^2 = 0$ is $y = \frac{1}{3x+C}$, where C is an arbitrary constant, and $y = 0$.