

Quiz 2 (with Solutions); October 15

(1) Indicate the correct option among the choices provided below.

Suppose f, g, h are functions that are continuous and have continuous first and second derivatives on the interval $I = (-\infty, \infty)$. Suppose $W(f, g, h)(1) = 0$.

Then:

- (a) This implies that the functions f, g, h are linearly independent on I .
- (b) This implies that the functions f, g, h are linearly dependent on I
- (c) None of the above.

Answer: (c).

(2) Consider the differential equation:

$$y^{(5)} - 3y'' + y' - 2y = 1$$

on the interval $I = (-\infty, \infty)$.

Indicate the correct option among the following choices:

- (a) This equation has exactly one solution on I .
- (b) This equation has the property that for any two solutions y_1, y_2 the function $y = y_1 + y_2$ is also a solution.
- (c) This equation has infinitely many solutions on I .

Answer: (c).

Note that the right-hand side of the equation is 1 rather than 0 so choice (b) is incorrect.

(3) Find the general solution of the equation $y'' + 2y' + 5y = 0$

Solution.

The characteristic equation is $r^2 + 2r + 5 = 0$. Using the quadratic formula, we get:

$$r = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i.$$

Therefore the general solution is

$$y = c_1 e^{-x} \cos(2x) + c_2 e^{-x} \sin(2x).$$