Quiz 5, December 5, 2003 (Solutions)

Problem 1. In this problem choose the correct option from the choices provided (no explanations are necessary).

Consider the system

$$\begin{cases} y_{tt} = 4y_{xx} \quad 0 < x < 3, \\ y(0,t) = y(3,t) = 0, \\ y(x,0) = f(x), \quad 0 < x < 3 \\ y_t(x,0) = 0, \quad 0 < x < 3. \end{cases}$$

Then:

(1) The system has a solution of the form $y = \sum_{n=1}^{\infty} A_n \cos \frac{4\pi nt}{3} \sin \frac{\pi nx}{3}$.
(2) The system has a solution of the form $y = \sum_{n=1}^{\infty} A_n \sin \frac{4\pi nt}{3} \sin \frac{\pi nx}{3}$.
(3) The system has a solution of the form $y = \sum_{n=1}^{\infty} A_n \sin \frac{2\pi nt}{3} \sin \frac{\pi nx}{3}$.
(4) None of the above.

Answer: (4) In this problem $a = \sqrt{4} = 2, L = 3$ and the correct form for $y(x,t)$ is in fact $y = \sum_{n=1}^{\infty} A_n \cos \frac{2\pi nt}{3} \sin \frac{\pi nx}{3}$.

Problem 2.

Find the solution $y = y(x,t)$ of the following system:

$$\begin{cases} y_{tt} = 5y_{xx} \quad 0 < x < \pi, \\ y(0,t) = y(\pi,t) = 0, \\ y(x,0) = 3\sin 2x \quad 0 < x < \pi, \\ y_t(x,0) = 0, \quad 0 < x < \pi. \end{cases}$$

Solution.

Here $a = \sqrt{5}, L = \pi$. Therefore

$$y = \sum_{n=1}^{\infty} A_n \cos \sqrt{5}nt \sin nx,$$

where

$$\sum_{n=1}^{\infty} A_n \sin nx = 3\sin 2x \text{ for } 0 < x < \pi.$$

Hence we have $A_2 = 3$ and $A_n = 0$ for $n \neq 2$. Thus

$$y = \sum_{n=1}^{\infty} A_n \cos \sqrt{5}nt \, \sin nx = 3\cos 2\sqrt{5}t \, \sin 2x.$$