

Some Review Problems

Problem 1. Show that the functions

$$f(x) = \begin{cases} 1 & x \text{ rational,} \\ -1, & x \text{ irrational} \end{cases}$$

and $g(x) \equiv 3$ are linearly independent on the interval $(-\infty, \infty)$.

Problem 2.

Find all the separated solutions of the system:

$$\begin{cases} u_{xx} + u_{yy} = 0, & 0 < x < \pi, 0 < y < \pi, \\ u(0, y) = u(\pi, y) = 0, & 0 < y < \pi \\ u(x, \pi) = 0, & 0 < x < \pi. \end{cases}$$

Note: There was a misprint in this problem in the printout handed out in class.

Problem 3.

Solve the system:

$$\begin{cases} y_{tt} = 25y_{xx}, & 0 < x < 3, t > 0, \\ y(0, t) = y(3, t) = 0, & t > 0 \\ y(x, 0) = 0, & 0 < x < 3 \\ y_t(x, 0) = 10 \sin 2\pi x, & 0 < x < 3. \end{cases}$$

Problem 4.

An object of mass $m = 1\text{kg}$ is attached to a spring with Hooke's constant $k = 4\text{N/m}$ and is acted on by a 2π -periodic force $F(t)$ Newtons where $F(t) = 1$ for $0 < t < \pi$ and $F(t) = -1$ for $-\pi < t < 0$.

Determine whether or not pure resonance occurs.

Problem 5.

Let $f(x)$ be a 6-periodic function such that $f(x) = x^2 - x$ for $-3 < x < 3$. Let a_n, b_n be the general Fourier series coefficients of $f(x)$. Find

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{46\pi n}{3} + b_n \sin \frac{46\pi n}{3} \right)$$

and

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \pi n + b_n \sin \pi n).$$

Problem 6.

Find the Fourier Sine Series of the function $f(x)$ defined on the interval $0 < x < 3$ as

$$f(x) = \begin{cases} 1 & 0 < x \leq 2, \\ 5, & 2 < x < 3. \end{cases}$$

Problem 7.

Find the general solution of the following equation:

$$y'' - 6y' + 9y = x^2 e^{3x} + \cos x.$$

Note: This is a pretty long problem!

Problem 8.

Let $y(x, t)$ be the solution of the system:

$$\begin{cases} y_{tt} = 4y_{xx}, & 0 < x < 1, t > 0, \\ y(0, t) = y(1, t) = 0, & t > 0 \\ y(x, 0) = x^2, & 0 < x < 1 \\ y_t(x, 0) = 0, & 0 < x < 1. \end{cases}$$

Using d'Alambert's method find the precise value of $y(1/2, 10)$.