Quiz 5 April 26, 2004 (SOLUTION)

Problem 1.

Solve the following heat equation problem:

$$\begin{cases} u_t = 2u_{xx}, & 0 < x < 4, t > 0 \\ u(0,t) = u(4,t) = 0 \\ u(x,0) = 5\sin(\pi x), & 0 < x < 4. \end{cases}$$

Solution.

In this problem we have $k = 2$, $L = 4$. Therefore by Theorem 1 on page 613

$$(\dagger) \qquad u(x,t) = \sum_{n=1}^{\infty} b_n exp(-\frac{\pi^2 n^2 kt}{L^2}) \sin\frac{\pi n x}{L} = \sum_{n=1}^{\infty} b_n exp(-\frac{2\pi^2 n^2 t}{16}) \sin\frac{\pi n x}{4}.$$

Where b_n are the Fourier Sine Series Coefficients of $f(x) = 5\sin(\pi x)$, $0 < x < 4$. Thus

$$\sum_{n=1}^{\infty} b_n \sin\frac{\pi n x}{4} = 5\sin(\pi x) = 5\sin(\frac{4\pi x}{4}), \quad 0 < x < 4,$$

gives us $b_4 = 5$ and $b_n = 0$ for $n \neq 4$. Hence from $(\dagger)$ we get

$$u(x,t) = 5\, exp(-\frac{2\pi^2 4^2 t}{16}) \sin\frac{4\pi x}{4} = 5\, exp(-2\pi^2 t)\sin(\pi x).$$