Definitions and Notations of SMSG and Neutral Geometry

(a) A point C is said to lie *between* points A and B, denoted $A - C - B$, if A, B, C are distinct and $d(A, B) = d(A, C) + d(C, B)$.

(b) For two distinct points A, B a *line segment* $\overline{AB}$ consists of the points A, B and of all points C on the unique line passing through A and B such that $A - C - B$. The points A and B are called the *endpoints* of $\overline{AB}$.

(c) For two distinct points A, B the unique line through A and B is denoted $\overleftrightarrow{AB}$.

(d) For two distinct points A, B the *ray* $\overrightarrow{AB}$ consists of the points A, B and of all the points C on the line $\overleftrightarrow{AB}$ such that $A - C - B$ or $A - B - C$.

(e) A set of points R is called *convex* if for any two distinct points A, B in R the segment $\overline{AB}$ is contained in R.

(f) If l is a line in a plane P then by SMSG Postulate 9 the complement of l in P is a union of two convex disjoint sets H_1 and H_2 such that for any points $A \in H_1$ and $B \in H_2$ the segment $\overline{AB}$ intersects l at a single point. The sets H_1 and H_2 are called *half-planes* of P determined by l.

(g) If A, B, C are three distinct non-colinear points, then the *angle* $\angle BAC$ consists of the points on rays $\overrightarrow{AB}$, $\overrightarrow{AC}$ and of all points D that lie between a point on $\overrightarrow{AB}$ and a point on $\overrightarrow{AC}$. The rays $\overrightarrow{AB}$, $\overrightarrow{AC}$ are called the *sides* of $\angle BAC$ and the point A is called the *vertex* of $\angle BAC$.

(h) An angle is *obtuse* if it has measure more than $90°$. An angle is *acute* if it has measure less than $90°$. An angle is *right* if it has measure $90°$.

(i) Two angles are *adjacent* of they have a common side and their intersection is equal to that side.

(j) Two angles are *vertical* if they have a common vertex, their intersection is equal to that vertex and if the sides of each angle can be ordered in such a way that the union of the first sides of these angles is a line and the union of the second sides of these angles is a line.

(k) Two angles are *supplementary* if the sum of their measures is $180°$. Two angles are *complementary* if the sum of their measures is $90°$.

(l) The midpoint of a line segment is a point on the segment that is equidistant from the endpoints of the segment.

(m) A *bisector* of an angle $\angle BAC$ is a ray $\overrightarrow{AD}$ that is contained in $\angle BAC$ and such that $m\angle BAD = m\angle DAC = \frac{1}{2}m\angle BAC$.

(n) Two lines l_1, l_2 are said to be *perpendicular* if they intersect at a point A such that for any point B on l_1 and any point C on l_2 such that $B \neq A$ and $C \neq A$ the angle $\angle BAC$ is right.

(o) If A, B, C are three non-colinear points then a *triangle* $\triangle ABC$ is the intersection of the angles $\angle ABC$, $\angle BAC$ and $\angle CBA$. The points A, B, C are called *vertices* of $\triangle ABC$ and the segments $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ are called the *sides* of $\triangle ABC$.

(p) Let $A_1, A_2, A_3, \ldots A_n$ (where $n \geq 4$) be n distinct points in the plane.

Suppose that:

(1) No three distinct points in the list $A_1, A_2, A_3, \ldots A_n$ are colinear; and
(2) For each $i = 2, \ldots, n-2$ the intersection of the triangles $\triangle A_1 A_i A_{i+1}$ and $A_1 A_{i+1} A_{i+2}$ is equal to the segment $\overline{A_1 A_{i+1}}$; and
(3) The union of the triangles $\triangle A_1 A_2 A_3, \triangle A_1 A_3 A_4, \ldots \triangle, A_1 A_{n-1} A_n$ is convex.

Then a *polygon* $P(A_1, \dots, A_n)$ is the union of triangles

$$\triangle A_1 A_2 A_3, \triangle A_1 A_3 A_4, \dots, \triangle A_1 A_{n-1} A_n.$$

The points $A_1, \dots, A_n$ are called the *vertices* of $P(A_1, \dots, A_n)$ and the segments $\overline{A_1 A_2}, \overline{A_2 A_3}, \dots, \overline{A_{n-1} A_n}, \overline{A_n A_1}$ are called the *sides* of $P(A_1, \dots, A_n)$.

(q) The *interior* of an angle (triangle, polygon) consists of all those points of the angle (triangle, polygon) that do not lie on its sides.