Solutions to H/wk 1

Ch 1.2 no. 5. We will prove that there are exactly 10 y's in the system.

Proof. By Axiom 1 there are precisely five x's in the system. By Axiom 2 any two distinct x's lie on an exactly one y and by Axiom 3 each y is on exactly two x's. Thus every unordered pair of distinct x's determines a unique y and every y arises in this way.

There are $(5 \cdot 4)/2 = 10$ ways to choose an unordered pair from the collection of five x's. Therefore there are 10 y's in the system. □

Ch 1.2 no. 6. We will now prove that any two distinct y's have at most one x on both.

Proof. Suppose not, so that for some two distinct y's there are at least two distinct x's that lie on both of these y's. Take two of these x's. Then these two x's have two distinct y's on both of them. This is impossible by Axiom 2. □

Ch 1.2 no. 8.

We will prove that there exist exactly four distinct y's on each x.

Proof. Take any x in the system and denote it x_0. By Axiom 1 there are exactly four x's in the system different from x_0. Axiom 2 implies that for each one of these four x's there is exactly one y that is on both x and x_0. By Axiom 3 all four of these y's must be distinct.

The result of no. 6 (proved above) and Axiom 3 imply that any y with x_0 on it must be equal to one of the four y's described above.

Thus there are exactly four distinct y's on x_0. Since x_0 was arbitrary, this proves the claim. □

Ch 1.2 no. 15. Consider a pentagon with vertices A, B, C, D, E in the plane, as shown in the figure below. These vertices will stand for x's and the segments joining each pair of distinct vertices will stand for y's, with the standard geometric interpretation of the term "on". This provides a valid model for the axiomatic system for exercise no. 5.