Partial Solutions to H/wk 11

Ch 6.4 no. 6. Prove that if a triangle $\triangle ABC$ is partitioned into a triangle and a quadrilateral by a line, then the defect of $\triangle ABC$ is equal to the sum of the defects of the two component pieces.

Solution.

Suppose a line l partitions $\triangle ABC$ into triangle $\triangle RSB$ and quadrilateral ARC, as shown in the figure below. Then

$$\begin{aligned} d(\triangle ABC) &= 180^\circ - m\angle 1 - m\angle 6 - m\angle 7 \\ d(\triangle RBS) &= 180^\circ - m\angle 1 - m\angle 2 - m\angle 3 \\ d(ARSC) &= 360^\circ - m\angle 6 - m\angle 4 - m\angle 5 - m\angle 7. \end{aligned}$$

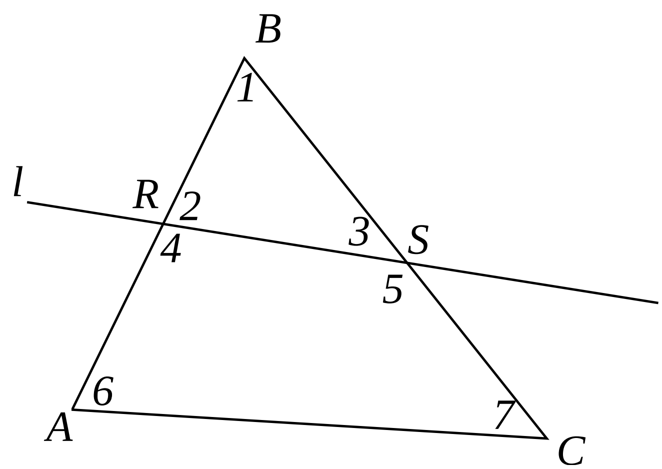

FIGURE 1. Figure for Problem no. 6 Ch. 6.4

Adding the last two equalities and taking intoi account that $m\angle 2+m\angle 4 = 180^\circ$, $m\angle 3 + m\angle 5 = 180^\circ$, we get:

$$\begin{aligned} &d(\triangle RBS) + d(ARSC) = \\ &= 180^\circ + 360^\circ - \big(m\angle 1 + m\angle 2 + m\angle 3 + m\angle 6 + m\angle 4 + m\angle 5 + m\angle 7\big) = \\ &540^\circ - (m\angle 1 + m\angle 6 + m\angle 7 + 180^\circ + 180^\circ) = 180^\circ - (m\angle 1 + m\angle 6 + m\angle 7) = \\ &= d(\triangle ABC) \end{aligned}$$

as required.

Ch 6.4 no. 7.

Suppose $\triangle ABC$ and $\triangle DFE$ are equilateral triangles in hyperbolic geometry such that $AB = 2DE$. Prove that $m\angle CAB < m\angle EDF$.

Solution.

Suppose not, that is $m\angle CAB \geq m\angle EDF$. If $m\angle CAB = \angle EDF$ then by Theorem 6.4.5 the triangles $\triangle ABC$ and $\triangle DEF$ are congruent, which contradicts the fact that $AB \neq DE$. Thus assume $m\angle CAB > m\angle EDF$.

Let M be the midpoint of AB so that $BM = DE$. Since $m\angle CAB > m\angle EDF$, there is a point Q is the interior of $\triangle ABC$ such that $\angle MBQ \cong \angle DEF$ and $BQ = EF$. Hence by the SAS axiom $\triangle MBQ \cong \triangle DEF$.

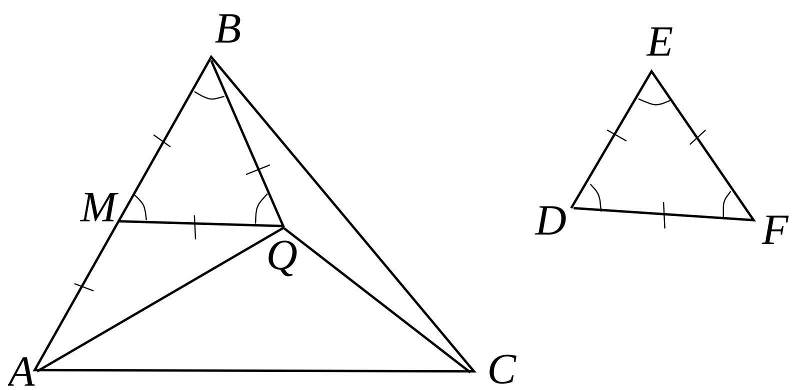

FIGURE 2. Figure for Problem no. 7 in Ch. 6.4

Draw segments $\overline{AQ}$ and $\overline{QC}$. By Theorem 6.4.6

$$d(\triangle ABC) = d(\triangle MBQ) + d(\triangle BQC) + d(\triangle AQC) + d(\triangle MQA).$$

Since by Theorem 6.4.1 all triangles in hyperbolic geometry have positive defects, the above equality implies

$$d(\triangle ABC) > d(\triangle MBQ) = d(\triangle EDF).$$

On the other hand, since the triangles $\triangle ABC$ and $\triangle EDF$ are equilateral, we have

$$d(\triangle ABC) = 180^\circ - 3m\angle CAB > d(\triangle EDF) = 180^\circ - 3m\angle EDF$$

and hence

$$m\angle CAB < m\angle EDF,$$

which contradicts our assumptions.

Ch 6.4 no. 16.

The defect of every triangle in Euclidean geometry is 0° since by Theorem 3.6.18 in Euclidean geometry every triangle has angle sum 180°.

The statement of Theorem 6.4.14 is not valid in the Euclidean geometry. Indeed, all triangles in Euclidean geometry have the same defect (namely 0°) but there do exist two inequivalent triangles (think about why).