

## Solutions to H/wk 2

### Ch 2.2 no. 2.

No, not all triangles are isosceles. The hidden assumption (which actually never holds) in Figure 2.2.3 was that a perpendicular bisector of the side  $BC$  and the bisector of the angle  $\angle BAC$  intersect at a unique point inside the triangle  $\triangle ABC$ .

### Ch 2.2 no. 4.

- (a) Two lines are *parallel* if there is no point that is on both of them.
- (c) For two distinct points  $A, B$  the *line segment*  $\overline{AB}$  consists of the points  $A, B$  and all points  $C$  such that  $C$  is between  $A$  and  $B$ .
- (b) Two lines  $l_1, l_2$  are *perpendicular* if they intersect at a single point  $C$  and there are two distinct points  $A, B$  on  $l_1$  such that  $C$  is between  $A$  and  $B$ , the segments  $\overline{AC}$  and  $\overline{BC}$  are congruent and such that for any point  $D$  on  $l_2$  the segments  $\overline{AD}$  and  $\overline{BD}$  are congruent.
- (d) For two distinct points  $A, B$  the *ray*  $\overrightarrow{AB}$  consists of the points  $A, B$  and all points  $C$  such that either  $C$  is between  $A$  and  $B$  or  $B$  is between  $A$  and  $C$ .
- (e) For two distinct points  $A, B$  the *circle* with center  $A$  passing through  $B$  is the set of all points  $C$  such that the segment  $\overline{AC}$  is congruent to the segment  $\overline{AB}$ .
- (f) Suppose  $A, B, C, D$  are four distinct points such that there are lines  $l_1, l_2, l_3, l_4$  with the following properties:
  - (1) The line  $l_1$  is perpendicular to  $l_2$  and  $l_4$ ;
  - (2) The line  $l_3$  is perpendicular to  $l_2$  and  $l_4$ ;
  - (3) The segment  $\overline{AB}$  is contained in  $l_1$ , the segment  $\overline{BC}$  is contained in  $l_2$ , the segment  $\overline{CD}$  is contained in  $l_3$  and the segment  $\overline{AD}$  is contained in  $l_4$ .

Then the *square with vertices*  $A, B, C, D$  consists of all points on the segments  $\overline{AB}, \overline{BC}, \overline{CD}, \overline{AD}$  and of all points  $Q$  that lie between a point on  $\overline{AB}$  and a point on  $\overline{CD}$ .

### Ch 2.2 no. 7.

Euclid's proof of the SAS principle is not valid since it assumes the ability to "move" objects in the plane. There is nothing in Euclid's axiomatic system that guarantees the ability to do so or defines what "moving" objects means.

### Ch 2.2 no. 8.

Statements (a), (b), (c), (d) can be used to show that Euclid's axiomatic system is not complete.

### Ch 2.2 no. 13.

The hidden assumption is that all triangles have the *same* sum of the measures of the interior angles.