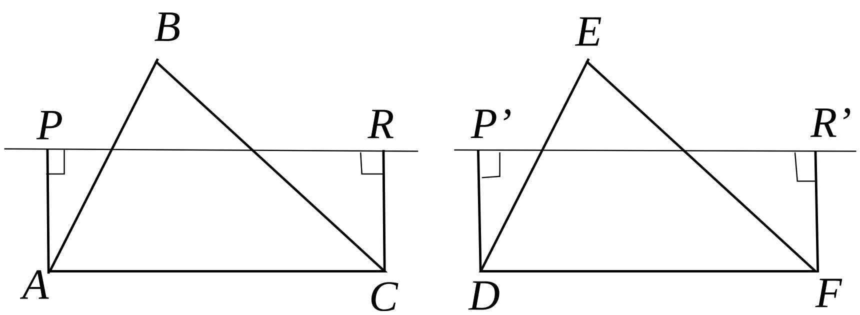
Math 302, Section B1; April 23, 2003
Quiz 10 (with Solution)

Problem 1.

Prove that in hyperbolic geometry if triangles $\triangle ABC$ and $\triangle DEF$ have the same defect and if $AC = DF$ then the associated Saccheri quadrilaterals for $\triangle ABC$ and $\triangle DEF$ based on $\overline{AC}$ and $\overline{DF}$ are congruent.

Solution.

Let $ACRP$ be the associated Saccheri quadrilateral of $\triangle ABC$ and let $DFR'P'$ be the associated Saccheri quadrilateral of $\triangle DEF$. Thus $\overline{AC}$ is the summit of $ACRP$ and $\overline{DF}$ is the summit of $DFR'P'$.



By construction of the Saccheri quadrilateral associated to a triangle (see page 319) $d(\triangle ABC) = d(ACRP)$ and $d(\triangle DEF) = d(DFR'P')$. By assumption $d(\triangle ABC) = d(\triangle DEF)$ and therefore $d(ACRP) = d(DFR'P')$. The defect of a Saccheri quadrilateral is equal to $360°$ minus the angle sum, that is, $180°$ minus twice the measure of the summit angle. Thus $d(ACRP) = d(DFR'P')$ implies that the summit angles of $ACRP$ and $DFR'P'$ are congruent. Since $ACRP$ and $DFR'P'$ also have congruent summits, Theorem 6.4.10 implies that $ACRP \cong DFR'P'$.